**H. E. ROSE**
**R. M. E. SULLIVAN**

# BALL
# TUBE
# AND
# ROD
# MILLS

$6.00

## BALL TUBE AND ROD MILLS

H. E. Rose
AND
R. M. E. Sullivan

This book throws new light on the internal mechanics of mills. It covers among others the following aspects: Ball, tube, rod, pebble, batch, grate-discharge, trunnion-overflow, hardinage and airswept mills; open- and closed-circuit grinding; dry and wet milling; contamination of products. Single ball in mill; cascading; cataracting; equilibrium surface of charge, etc. The power to drive a mill. Perfect crystals; heterogeneous bodies; crushing a single particle; scale effects; effects of impact velocity. Dimensional analysis; rate of grinding. Effect of pulp consistency on preferential grinding; influence of shape of grinding media; size distribution of grinding media; rate of flow through continuous mills. Surging; lifters; vibration frequency spectrum; destruction of gearing. Rose's theory of metal wear; factors affecting ball wear; influence of corrosion. The role of additives in milling.

# A TREATISE ON THE INTERNAL MECHANICS OF BALL, TUBE AND ROD MILLS

*by*

H. E. ROSE
D.Sc.(Eng.), Ph.D.

AND

R. M. E. SULLIVAN
B.Sc.(Eng.)

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# FOREWORD

*by*

PROFESSOR DR.-ING. ROBERT MELDAU

IT is with considerable pleasure that I write this Foreword to a book, written by Dr. Rose, devoted to the analysis of the internal processes of the family of mills which have freely moving grinding media.

Dr. Rose is well-known to the Fachgruppe "Staubtechnik" of the Verein Deutscher Ingenieure, for his lectures in Germany on his researches into milling and into the general properties of powdered materials and from my knowledge of him and of the researches which are being carried out under his direction, I am sure that Dr. Rose and Mr. Sullivan are well equipped to write on the subject.

The need for a book devoted to such an analysis is evident when the great number of "tumbling" mills used in industry, their low efficiency and the great resistance which they have offered to any substantial improvement, is called to mind. Thus, this book tackles a problem to which much thought has been devoted during at least a century and to the solution of which every advance will be welcome.

Dr. Rose uses, with convincing results, the method of dimensional analysis as a preferred approach to his problems and it appears that the statistical behaviour of fine particles lends itself to such treatment, to a remarkable extent.

To speak in milling terms; may the mill of Dr. Rose's thoughts yield ever finer products for the use of those interested in this important subject! I believe that this book will provide improved theory and also furnish the means whereby those interested can obtain a deeper understanding of the internal mechanics of such mills.

ROBERT MELDAU

POLYTECHNISCHE HOCHSCHULE,
HANNOVER.

# PREFACE

As with the previous volumes in the same series, the object of the present work is to bring together the data relating to some part of the field of science dealing with powdered materials, in this case to the internal mechanics of the ball, tube and rod mills, to analyse these data in the light of the Authors' experience and researches and, as far as is possible with the present state of knowledge of the subject, to present the results in a systematized form.

Attention has been given to the ball, tube and rod mill because, although the results of numerous tests on these machines have been published, it is believed that no comprehensive analysis has yet been made. Thus, for example, although many investigations have been carried out to show the influence of the various parameters defining the characteristics of the mill and the charge, as far as is known there has previously been offered no formula for the calculation of the instantaneous rate of production of specific surface from a knowledge of these parameters. In Chapter 5, however, such a formula is provisionally presented.

It is suggested that the formula is, at present, subject to an error of, say, $\pm 50\,\%$, and so is to be regarded as giving a first approximation only to the desired result. Nevertheless, it is believed that the reduction of these data to such a formula is a considerable advance along the road to placing the subject on a firm foundation and that the filling in of details, with consequent improvement in accuracy, is largely a matter of well-conducted experiment. Even though the highest accuracy is lacking in the overall formula, the work is believed to be of some practical value since the influence of nearly every variable has been analysed in the light of the available data. Thus, for example, the mill operator will find information as to the probable effect of variation of, say, ball size upon the grinding characteristics and power consumption of a mill and so he may predict, with some confidence, the direction in which a contemplated change in the mill parameters is likely to alter the mill performance.

The reasons for restriction to the internal mechanics of the mill are several; these reasons being:

(1) Several books dealing with the general design and operation of the tumbling mill are in existence, e.g. Herr Carl Mittag's *Hartzerkleinerung*.
(2) As far as is known no book dealing with the internal processes in a mill is available.
(3) To have treated the problems of design and operation of mills, in addition to the present field, would have defeated one of the objects of the Authors, which was to produce a small volume at moderate price which would fill a gap in the literature on the subject. It was not the intention to write more on parts of the field which are already adequately covered.

The title "Internal Mechanics" has been chosen since only phenomena occurring in, or originating within, the shell of the mill have been treated. Thus, details such as classifiers, feed mechanisms, gearing and bearings have not been considered. The breakdown of gearing has been considered in conjunction with "surging", however, since it is the Authors' belief that gear failures often arise from vibrations which are initiated by the oscillation (surging) of the mill charge within the shell.

Similarly, the power required to drive a mill has been studied in detail, since the power demand is clearly controlled by the configuration of the charge within the mill and is so a matter of internal dynamics.

No apology is offered for the introduction of a short review of the present state of knowledge of comminution of solid bodies since a knowledge of the operations taking place within the mill cannot be most effectively applied without a background knowledge of the process for which the mill is required.

Similarly, the introduction of a chapter reviewing the subject of milling additives is justified on the grounds that these substances can greatly modify the behaviour of a material during comminution and so, even though it is not definitely known whether this action is physical, chemical or mechanical, it cannot be omitted from a book which purports to deal with the internal operations of the "tumbling" mill.

Even though final answers cannot yet be given to many of the problems encountered in the design and application of the ball, tube and rod mill, it is hoped that useful lines of treatment will be suggested to the reader.

Whilst it is fully realized that this work is not a text-book, in the sense that worked out and accepted theories are presented, nor is it a design manual in which is presented a collection of formulae and useful data for designer and operator, it is hoped that it will be of interest and value to the mill designer, to the operator, to the chemical engineer and to the general reader with an interest in mechanics and engineering.

The Authors wish to express their indebtedness to all the authors whose works have been cited, and in particular to Mr. John Gross and Messrs. Coghill and Devaney whose works have been quoted at great length, to the U.S. Bureau of Mines and to the American Institution of Mining and Metallurgical Engineers, for permission to reproduce extensive data from the various publications mentioned in the text.

Thanks are also due to all the manufacturers who have supplied illustrations and other data, their names being noted against the appropriate illustrations, for their assistance and permission to reproduce the material supplied, to the I.M.E. for permission to reproduce Figs. 3.2, 3.4, 3.10 and 3.11, to Professor Andreasen for permission to reproduce Fig. 4.11 and to Dr. J. M. Connor and the University of London for permission to reproduce Figs. 2.4 and 2.5.

Finally, our best thanks are due to Mr. J. F. Hinsley and Monsieur P. Fobelets, who have kindly made available the results of their researches which have extended over many years and upon whose mathematical analyses much of the treatment of the motion and configuration of the mill charge is based, for their personal assistance and advice. Finally, thanks are due to all whose writings have been consulted and to all who have contributed to the work by advice and criticism.

POWDER SCIENCE LABORATORY H. E. R.
KING'S COLLEGE, LONDON R. M. E. S.

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# LIST OF SYMBOLS

| | |
|---|---|
| $b$ | Diameter of particle |
| $d$ | Diameter of ball (or rod) |
| $D$ | Diameter of mill shell |
| $e$ | Base of natural logarithms |
| $e$ | Coefficient of restitution |
| $E$ | Energy necessary to change the surface of powder by unit amount |
| $E_m$ | Energy necessary to change the surface of metal ball by unit amount |
| $f$ | Coefficient of friction |
| $g$ | Acceleration due to gravity |
| $h$ | Hardness of material or Height of lifter |
| $H$ | Relative hardness (Hardness of ball ÷ Hardness of particle) |
| $H_h$ | Horizontal displacement of centre of gravity of mill charge |
| $J$ | Fractional ball filling of mill<br>$= \dfrac{\text{Volume of balls} + \text{Volume of space between balls}}{\text{Volume of mill}}$ |
| $k$ | A parameter or Specific rate of grinding |
| $K$ | A parameter |
| $L$ | Length of mill |
| $M$ | Weight of product per revolution |
| $n$ | Speed of rotation of shell in r.p.m. or Number of lifters |
| $N$ | Speed of rotation of shell |
| $N_c$ | Critical speed of rotation |
| $P$ | Power to drive the mill |
| $q$ | Quantity of balls in flight |
| $Q$ | Mill throughput (mass per unit time) |
| $r$ | Radius of element of charge from centre of rotation |
| $R$ | Radius of mill shell or Reduction ratio |
| $S$ | Specific surface (Surface per unit volume) |

| | |
|---|---|
| $S_t$ | Total surface |
| $t$ | Time of grinding |
| $T_r$ | Time of revolution of mill |
| $T_c$ | Time for element of charge to traverse circular path |
| $T_p$ | Time for element of charge to traverse parabolic path |
| $u$ | Diameter of exit from mill |
| $U$ | Pulp consistency (volume of solid to volume of liquid) |
| $V$ | Velocity of projection of an element of charge or Fractional filling of solid $= \dfrac{\text{Volume of powder} + \text{Volume of space between particles}}{\text{Volume of space between the balls}}$ |
| $W_b$ | Wear of a single ball |
| $W_t$ | Total wear of balls |
| $Z$ | Ratio of circulating load to total load |
| $\alpha$ | Angle of displacement of centre of gravity or Angle of projection |
| $\beta$ | Parametric angle |
| $\gamma$ | Angle of repose of charge |
| $\delta$ | Angle between rays of the equiangular spiral or Mean density of the mill charge |
| $\Delta$ | Semi-angle subtended by the charge at the centre of the mill |
| $\varepsilon$ | Parametric angle |
| $\eta$ | Angular velocity of mill shell (rad/sec) |
| $\theta$ | Angle of displacement of centre of gravity of mill charge from the vertical |
| $\lambda$ | Coefficient of friction of charge |
| $\mu$ | Absolute viscosity |
| $\nu$ | Kinematic viscosity |
| $\xi$ | Ratio: $\dfrac{\text{Weight of ball charge} + \text{Weight of powder charge}}{\text{Weight of ball charge}}$ |
| $\varrho$ | Density of grinding media or Radius of curvature of ball path |
| $\sigma$ | Density of powder |
| $\phi$ | Denotes "a function of" |
| $\psi$ | A parameter |
| $\omega$ | Angular velocity of precession of the mill charge or element of charge |

CHAPTER 1

# INTRODUCTION

In many industries the final product, or the raw material at some stage of the manufacturing process, is in powdered form and in consequence the rapid and cheap preparation of powdered materials is a matter of considerable economic importance.

In some cases the powdered material may be prepared directly; for example by precipitation from solution, a process which is used in the preparation of certain types of pigments and drugs, or by the vacuum drying of a fine spray of the material, a process which is widely adopted for the preparation of milk powder, soluble coffee extracts and similar products. Such processes are, however, of limited applicability and in by far the greatest number of industrial applications the powdered materials are prepared by the reduction, in some form of mill, of the grain size of the material having an initial size larger than that required in the final product. These processes for the reduction of the particle size of a granular material are known as "milling" or "grinding" and it appears that these names are used interchangeably, there being no accepted technical differentiation between the two.

The reasons for the grinding of industrial materials are numerous but the principal reasons may be summarized under the following headings:

(1) The liberation of an economically important material from the unrequired constituents of a mixture.

(2) The exposure of a large surface per unit mass of material in order to facilitate some chemical process.

(3) To reduce the material to the desired form of the final product.
(4) To satisfy market requirements.

Examples of the first two classes occur in mineral dressing, in which size reduction is used to liberate the desired ore from the gangue and also to reduce the ore to a form in which it presents a large surface to the leaching reagents.

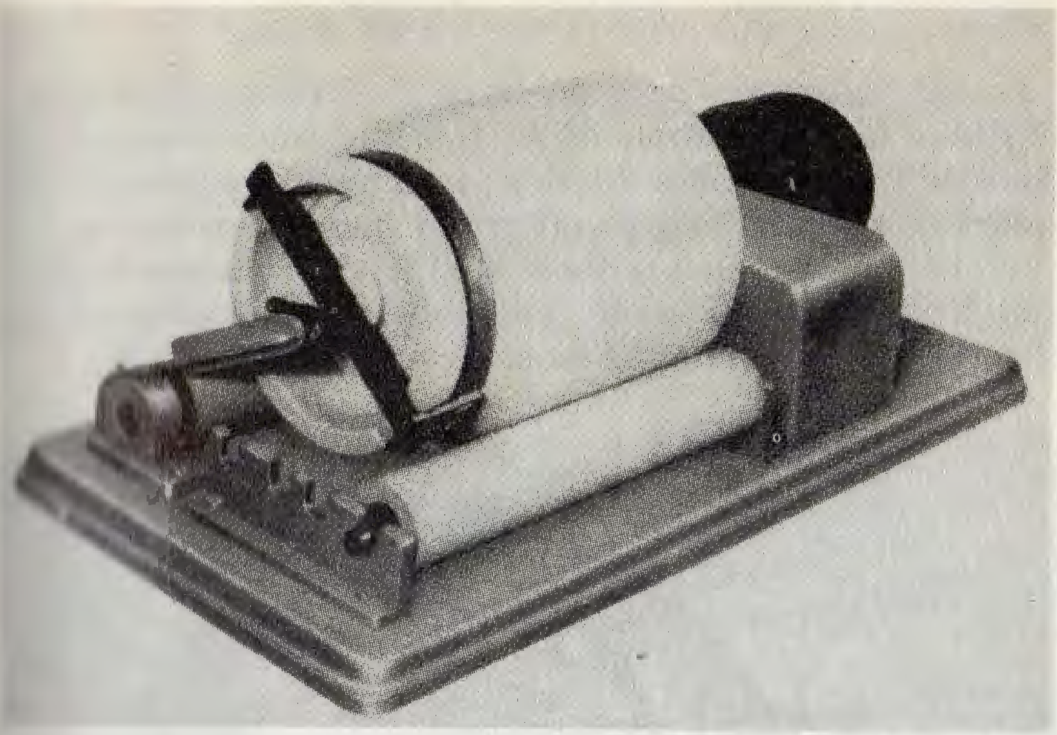
Under the third heading may be classed many medicinal and pharmaceutical products, foodstuffs, fertilizers, insecticides, etc., and under the fourth heading falls the size reduction of mineral ores, etc.; these materials often being reduced to particles of moderate size for ease in handling, storing and loading into trucks and into the holds of ships.

The quantity of powder to be subjected to such processes of size reduction varies widely according to the industries involved, for example in the pharmaceutical industries the quantitites involved per annum can be measured in terms of a few tons, or in the case of certain drugs, possibly a few pounds; whereas in the cement industry the quantities involved run into tens of millions of tons; the British cement industry alone having produced, in round figures, 12½ million tons of Portland Cement in 1955.

For the preparation of small quantitites of powder many types of mill are available but, even so, the ball mill is frequently used. For the grinding of the largest quantities of material however, the ball, tube or rod mill is used almost exclusively, since these are the only types of mill which possess throughput capacity of the required magnitude.

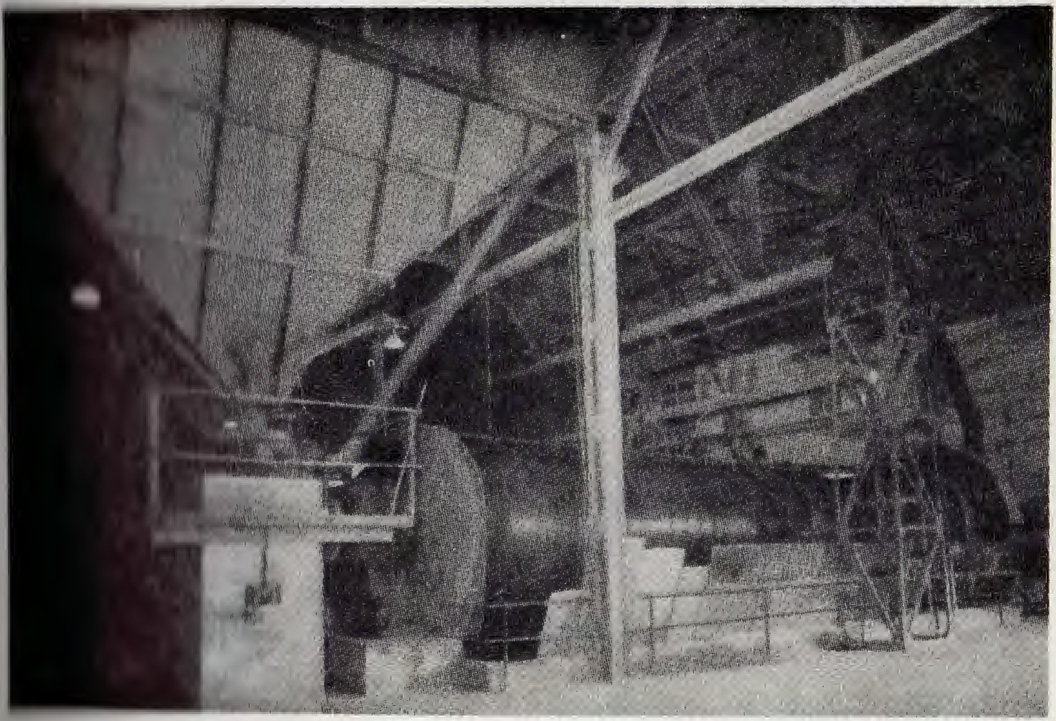
Thus, it is the extremely wide range of applications, and the great industrial importance, of the mills which constitute the "tumbling mill" family which forms the justification for the present work.

The great range of sizes covered by industrial ball mills is well exemplified by Fig. 1.1 and Fig. 1.2. In the first illustration is shown a laboratory batch mill of about 1-litre capacity, whilst in Fig. 1.2 is shown a tube mill used in the



(*Pascal Engineering Co. Ltd.*)

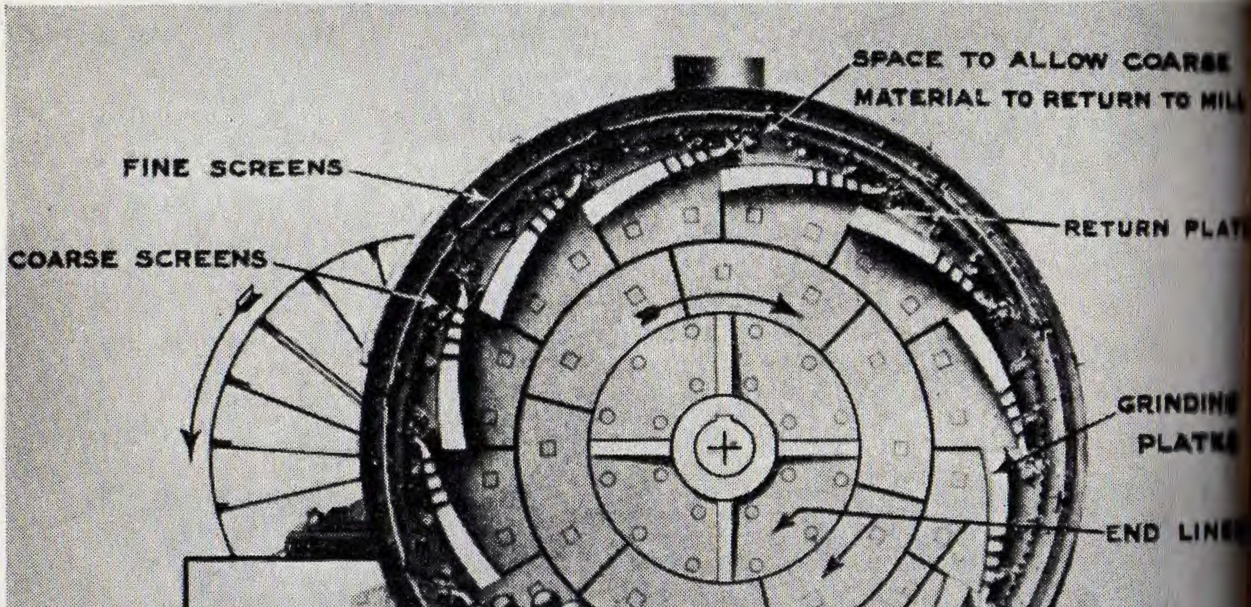
FIG. 1.1
Laboratory Batch Mill



(*Edgar Allen & Co. Ltd.*)

FIG. 1.2
Large Industrial Tube Mill

cement industry, the tube having a diameter of about 8 ft and length of about 45 ft. In Fig. 1.3 is shown a large ball mill, designed for the dry grinding of limestone, dolomite, quartz, refractory and similar materials; this type of mill being made in a series of sizes having diameters ranging from about 26 in. to 108 in., with the corresponding lengths of drum ranging from about 15 in. to 55 in.



(*Edgar Allen &*

FIG. 1.3
Industrial Ball Mill

At this point it is perhaps of value to study the nomenclature used in connection with the mills under consideration, but it must be emphasized that the lines of demarcation between the types to which the names are applied are not very definite.

The broadest classification appears to be:

(1) Ball mills.
(2) Tube mills.
(3) Rod mills.
(4) Pebble mills.

The term "ball mill" is usually applied to a mill in which the grinding media are bodies of spherical form ("balls") and in which the length of the mill is of the same order as the diameter of the mill body; in rough figures the length is, say, one to three times the diameter of the mill.

The "tube mill" is a mill in which the grinding bodies are spherical but in which the length of the mill body is greater in proportion to the diameter than is the case of the "ball mill"; in fact the length to diameter ratio is often of the order of ten to one.

The rod mill is a mill in which the grinding bodies are circular rods instead of balls, and, in order to avoid "tangling" of the rods, the length to diameter ratio of such mills is usually within the range of about 1·5 to 1 and 5 to 1.

The pebble mill is the name associated with a mill in which the grinding bodies are natural pebbles or possibly ceramic bodies; the general proportions of the mill being as for a "ball mill".

It will be noticed that the differentiation between ball mill and the tube mill arises only from the different length to diameter ratios involved, and not from any difference in fundamental principles. The rod mill, however, differs in principle in that the grinding bodies are rods instead of spheres, whilst a pebble mill is a ball mill in which the grinding bodies are of natural stone or of ceramic material.

The types of mill normally encountered can further be classified according to the method of feeding the raw material to, and the removal of the final product from the mill.

These classes are:

(1) Batch mills.
(2) Grate discharge mills.
(3) Trunnion overflow mills.
(4) Air-swept mills.

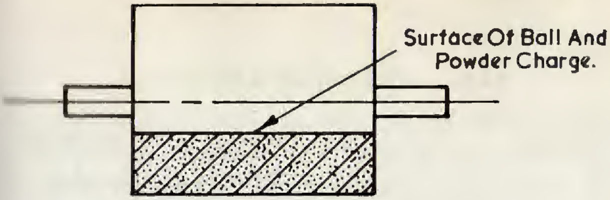
The principles of these mills are illustrated in Fig. 1.4 and Fig. 1.5.

As the name implies, in the batch mills, Fig. 1.4a, the charge of powder to be ground is loaded into the mill in a batch and, after the grinding process is completed, is removed in a batch. Clearly such a mode of operation can only be applied to mills of small or moderate sizes; say to mills of up to about 7 ft diameter by about 7 ft long.

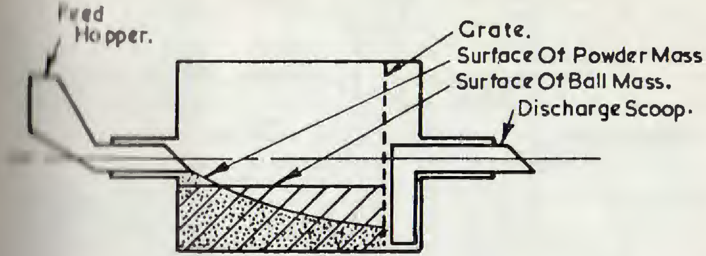
In the three other types of mill a continuous supply and removal of the powder is maintained, and these methods are applicable to mills of the largest sizes.

In the grate discharge mill, Fig. 1.4b, a diaphragm in the form of a grating confines the ball charge to one end of the mill and the space between the diaphragm and the other end of the mill houses a scoop for the removal of the ground material. The raw material is fed in through a hollow trunnion at the entrance end of the mill and during grinding traverses the ball charge; after which it passes through the grating and is picked up and removed by the discharge scoop or is discharged through peripheral ports. In this connection, it is relevant to mention that scoops are sometimes referred to as "lifters" in the literature. In the present work, the use of the term "lifter" will be confined to the description of a certain form of mill liner construction, fitted with "lifter bars" in order to promote the tumbling of the charge, which will be described in a later section.

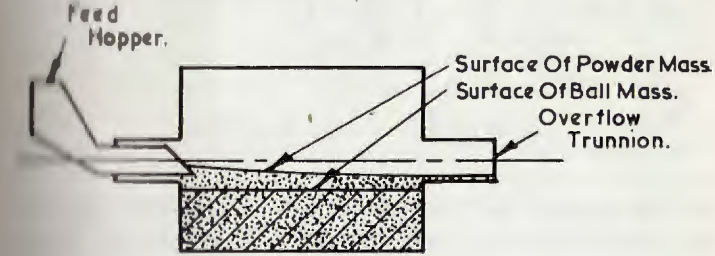
In the trunnion overflow mill, Fig. 1.4c the raw material is fed in through a hollow trunnion at one end of the mill



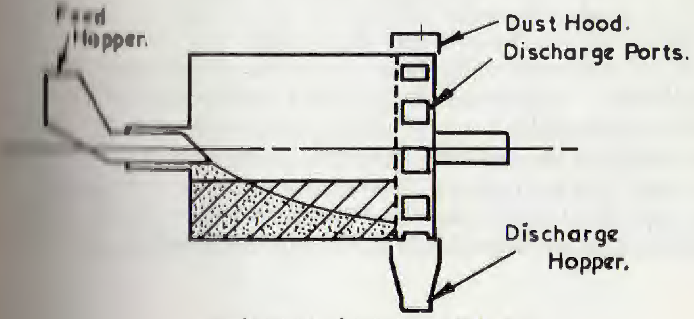
(a) Batch Mill.



(b) Grate Discharge Mill.

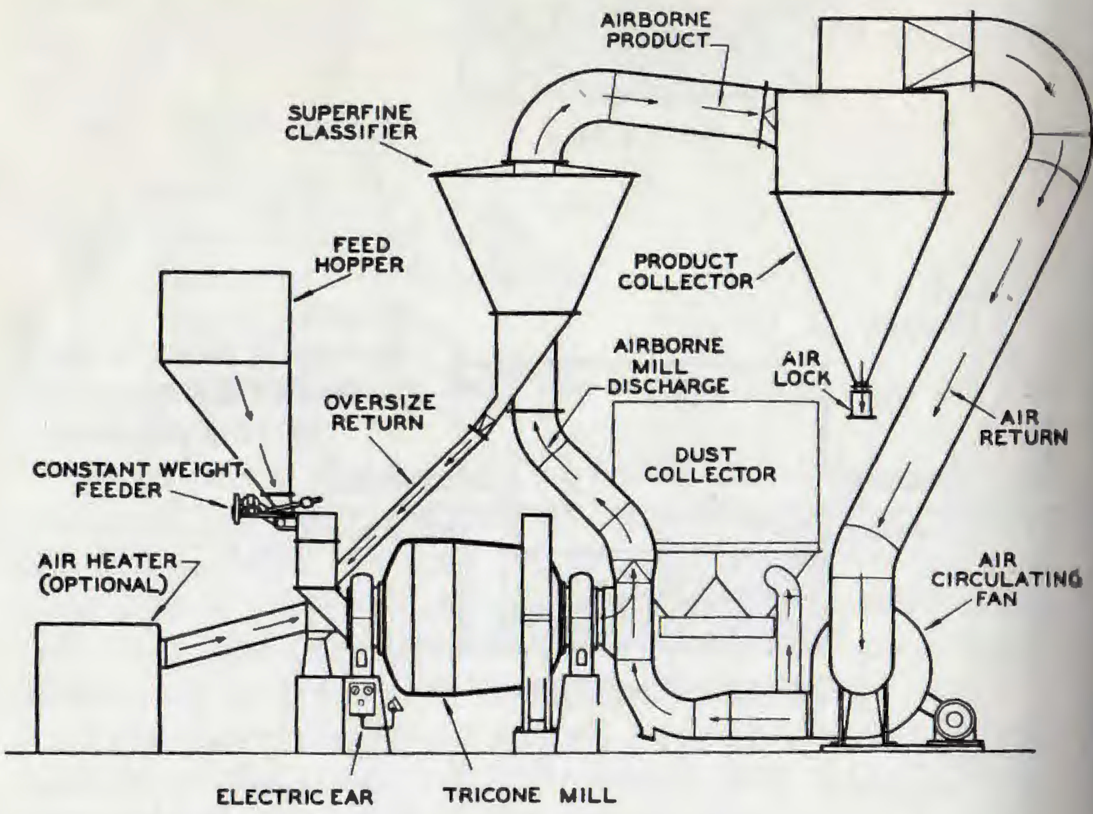


Trunnion Overflow Mill. (c)



Peripheral Discharge Mill (d)

Fig. 1.4



(*Hardinge Co. Inc.*)

Fig. 1.5

Layout of a typical closed-circuit plant

and the ground product "overflows" at the other end. In this case, therefore, the grating and discharge scoop are eliminated.

A variant of the grate discharge mill is shown in Fig. 1.4d, in which the discharge scoop is eliminated by the provision of peripheral discharge ports, with a suitable "dust" hood, at the exit end of the mill.

In an air-swept mill, the body of which does not differ significantly in general design from a trunnion overflow mill, a powerful stream of air is passed through the mill and removes the finer particles produced by the grinding process. This stream of powder-laden air is then passed through the classifier and the "oversize" material, rejected by the classifier, is returned to the mill with sufficient feed material to maintain the charge at the working level. The layout of such a closed-circuit plant is shown in Fig. 1.5.

The grinding in a batch mill, a grate discharge mill or trunnion overflow mill may be performed either “wet” or “dry”, but clearly in an “air-swept” mill the grinding must be carried out in the dry state.

In “wet” grinding the material to be ground is fed to the mill in the form of a “pulp” containing, perhaps, 50% by weight of water. In dry grinding, however, the granular material is fed into the mill in a sensibly dry condition; the only moisture present being the few percent normally held by the powder. Both the grate mill and the trunnion overflow mill may be operated in either of two distinct ways, known respectively as “open-circuit” operation and “closed-circuit” operation, and, furthermore, either of these methods of operation may be worked either “wet” or “dry”.

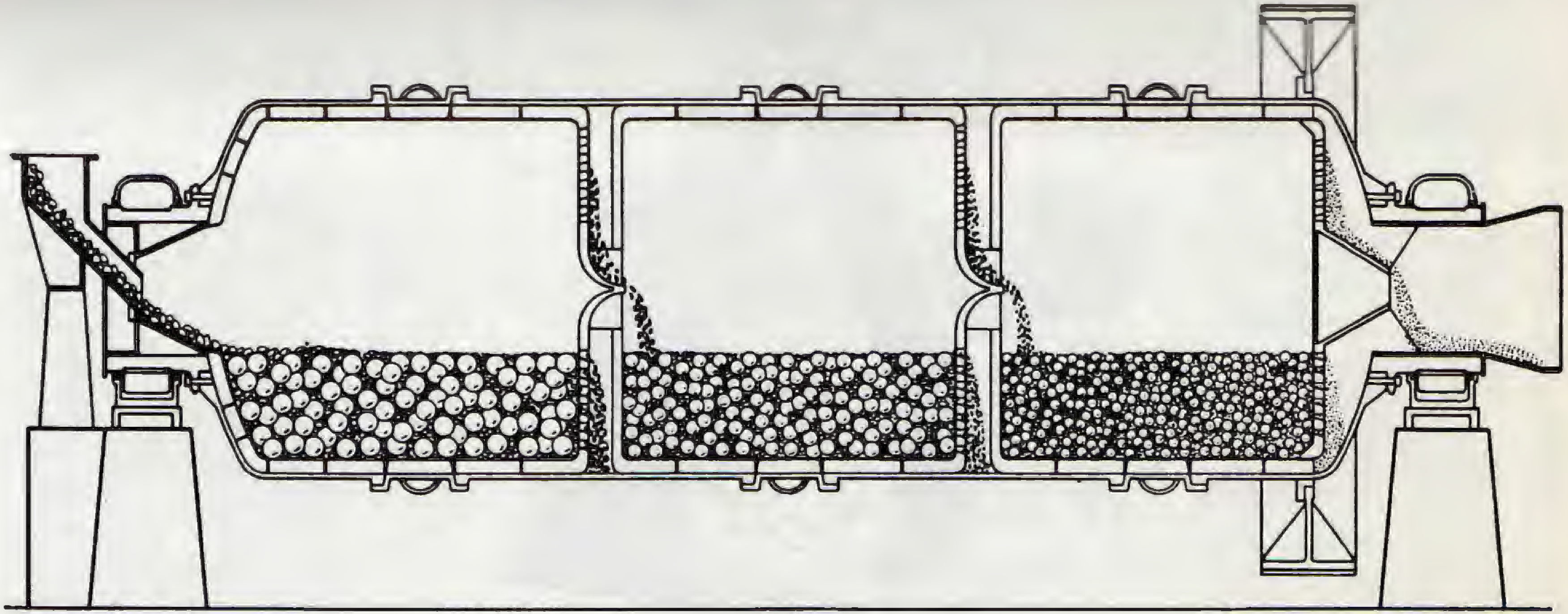
In open-circuit grinding the mill product is not subject to any classification, other than that which occurs within the mill by reason of selective grinding of the larger particles, and no part of it is returned to the mill for further treatment. In closed-circuit grinding, however, the product of the mill is conveyed to some form of classifier, and the material coarser than the required maximum size is returned to the mill feed; sufficient new feed material being added to maintain the charge within the mill at the operating level. For wet grinding a hydraulic classifier is used whilst for dry grinding an air separator, normally of the cyclone type, is necessary.

The “air-swept” method of operation of a mill eliminates the conveyor system which was often used in earlier types of plant to transport the product to the classifier and the oversize material from the classifier back to the mill. It is clear, however, that such a system can only be used successfully when the material undergoing treatment is sufficiently light, and the particles produced are sufficiently small, for the desired final product to be entrained in the air stream. Furthermore, the removal of the powder from the air stream, or its utilization whilst suspended in the stream must be practicable and, for these reasons, a typical application of systems of this type is in connection with the grinding of coal for pulverized fuel firing, since, in this case, the fuel may be burnt directly in the air stream which carries it.

Within the classes of mills enumerated above there are a number of variations; for example there occur in practice mills in which the shell is divided into a number of chambers by means of perforated diaphragms and it is arranged that the mean diameter of the balls in the various chambers shall decrease towards the discharge end of the mill; such an arrangement being shown in Fig. 1.6. The reason for this distribution of ball size is that, for optimum grinding conditions, the ratio of ball diameter to particle diameter should be approximately constant. In consequence smaller balls should be used for the later stages of the grinding process, where the powder is finer, and by the adoption of a number of chambers in each of which the mean ball diameter is suitably chosen an approximation is made towards the desired constancy in the ratio of the ball size to the particle size.

The problem of the optimum distribution of ball size within a mill will be dealt with more fully in a later chapter, but at this point it is relevant to mention a mill in which the segregation of the balls is brought about by an ingenious method; especially as the mill carries a distinctive name, even though no principles which place it outside the classification given previously are involved.

The Hardinge mill, Fig. 1.7, uses spheres as a grinding agent but the body is of cylindro-conical form and usually has a length to diameter ratio intermediate between those associated with the ball mill and the tube mill. The reason for this form of construction is that it is found that, during the operation of the mill, the largest balls accumulate at the large end of the cone and the smallest balls at the small end; there being a continuous gradation of size along the cone. If then the raw material is fed in at the large end of



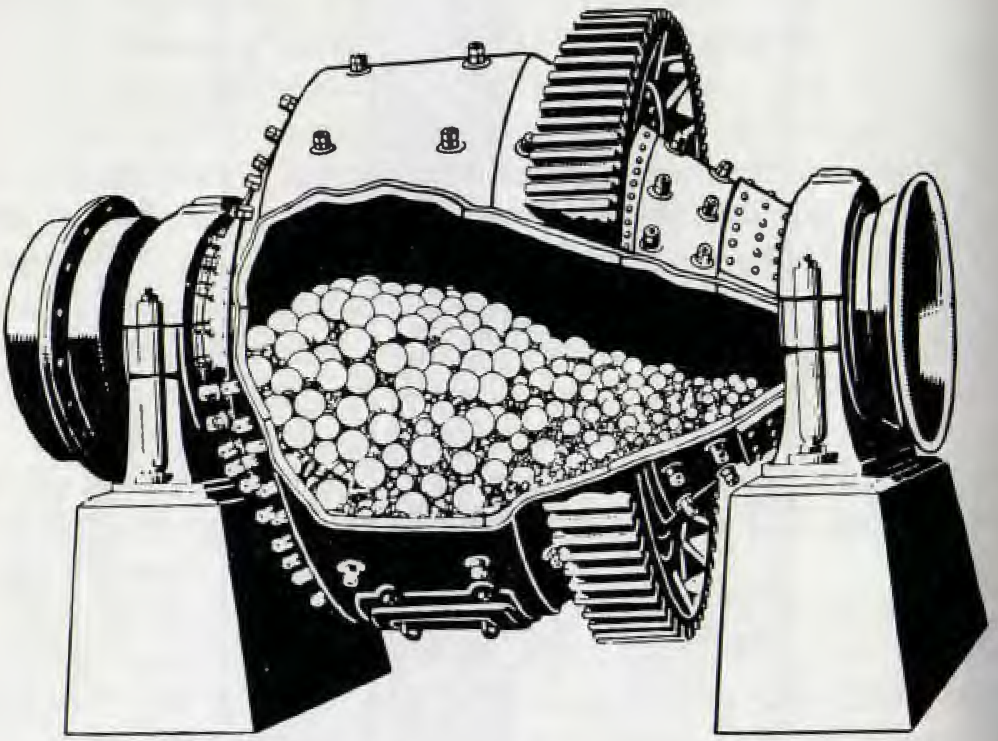
(*Hardinge Co. Inc.*

Fig. 1.6

Three-compartment tube mill

the mill and the ground product removed at the smaller end, the powder in its progression through the mill is ground by progressively small balls and in consequence the theoretical ideal of a constant ratio between ball size and particle size during grinding is, to some extent, attained.

The type of ball mill illustrated in Fig. 1.3, incorporates a peripheral discharge through fine screens lining the cylindrical part of the mill. Heavy perforated plates protect the screens from injury and act as a lining for the tumbling charge; sometimes also the fine screen is further protected by coarse screens mounted directly inside it. This type of mill, which is often known as the Krupp mill, is of interest since it represents a very early type of mill which, with modifications, has retained its popularity. The Krupp mill is particularly suited to the grinding of soft materials since the rate of wear of the perforated liners



(*Hardinge Co. Inc.*)

Fig. 1.7

The Hardinge Mill

is then not excessive. At this point it will perhaps be useful to discuss the factors upon which the choice between a ball, a tube or a rod mill depends.

When a mill is used as a batch mill, the capacity of the mill is clearly limited to the quantity which can be handled manually; furthermore the mill is, as far as useful work is concerned, idle during the time required for loading and unloading the machine: the load factor thus being adversely affected. Clearly then, there will be a considerable gain in throughput, a saving in handling costs and improved load factor, if the mill operation is made continuous by feeding the material into the mill through one trunnion and withdrawing it either through the other trunnion or through discharge ports at the exit end of the mill body.

Since, however, the flow of powder through the mill is now continuous, it is necessary that the mill body is of such a length that the powder is in the mill for a time sufficiently long for the grinding to be carried to the required degree of fineness. This, in general, demands a mill body of considerable length, or continuous circulation with a classifier, and it is increased length which gives rise to the "tube mill".

In the metallurgical industries very large tonnages have to be handled and, furthermore, an excess of fine material is undesirable since it often complicates subsequent treatment processes. In such applications a single-stage tube mill in circuit with a product classifier, by means of which the material which has reached optimum fineness is removed for transport to the subsequent processing and the oversize is returned to the mill for further grinding, is an obvious solution. Once continuous feed and a long mill body have been accepted, however, the overall grinding efficiency of the mill may be improved by fairly simple modifications.

As has already been mentioned; for optimum grinding conditions there is a fairly definite ratio of ball size to particle size and so the most efficient grinding process cannot be attained when a product with a large size range is present in the mill. If, however, a tube mill is divided into a number of compartments and the mean ball size of the grinding media decreases in each succeeding compartment; then the optimum ratio between ball size and particle size is more nearly maintained, and a better overall performance of the mill is achieved; this giving rise to the compartment mill shown in Fig. 1.6. The tube mill has the further advantage that, to some extent, the grinding characteristics of the mill are under control; for example, an increase in the size of the balls in the final chamber will reduce the rate of grinding of the finer fractions but will leave the rate of grinding of the coarser fractions sensibly unchanged and so the amount of coarse material in the final product will be reduced without any excessive overall increase in fineness.

The principal field of application of the rod mill is probably as an intermediate stage between the crushing plant and the ball mills, in the metallurgical industries. Thus, material between about 1-in. and 2-in. size may be reduced to about −6 mesh for feeding to the ball mills. Rod mills are, however, being used in closed circuit with a classifier to produce a product of less than about 48-mesh size, but such applications are unusual.

The choice between wet and dry milling is, in general, unimportant in small-scale milling but is a major technical problem when large-scale milling in the metallurgical industries is involved. On purely mechanical grounds it is difficult to see any great difference in fundamental principles between wet and dry milling, since dry milling may be regarded as wet milling with a fluid having the viscosity and density of air, whereas with wet milling the fluid is a liquid. A slight difference of condition between the two cases, however, is that with dry milling the fluid (air) fills the whole of the mill shell not occupied by the balls, whereas with wet milling the liquid occupies only a part of the mill volume not occupied by the solid charge. This difference would be expected to have some slight effect on the motion of the ball charge but the effect would be expected to be so small as to be practically negligible.

An examination of the literature shows, however, that the actual difference between the results of wet and dry milling are often very large; for example the power to drive a tube mill in which a given material is being wet ground, may be as much as 30 % less than that of a similar mill dry grinding. Furthermore the throughput of a continuous feed mill is greater in wet grinding; probably due to the better flow properties of a pulp in comparison with a dry material. It appears, however, that the rate of ball and liner wear is greater in wet milling so the decision between wet and dry operation is, in cases where the choice is permitted, a matter of balancing the economic gain arising from reduced power demand and increased throughput against the increased cost of replacement of the ball charge and the mill liners and the loss of production during the shut down period necessary for such replacements to be made.

Clearly in certain applications, this choice is not permitted since, for example, the grinding of the raw mix in a cement plant may be carried out either wet or dry but the grinding of the cement can, from the nature of the material, only be carried out dry.

A further important point of difference between wet and dry milling is that wet milling allows the easy introduction of surface active media for the reduction of the energy required to produce unit new surface or for the inhibition of aggregation of the fine particles. It is theoretically possible to introduce such substances in gaseous or vapour form into a dry mill but, in practice, such methods have not been sufficiently successful to justify wide adoption, and, in fact the only practicable method of introducing such materials is wet milling.

Consideration can now well be given to some of the factors upon which the choice of the type of mill, or the method of operation of a given mill, for the grinding of a specified material will depend. In the first place it is perhaps of interest to note that the problems encountered in the preparation of powdered materials are not necessarily reduced when the quantity of material to be processed is small. As an example of this, in the preparation of materials such as Portland Cement, which sell at relatively low prices, the problem is mainly one of obtaining a final product having the required size characteristics at a minimum total cost per ton and, in this case, questions of the the contamination of the material, during processing, scarcely arise.

In the grinding of drugs and cosmetic preparations, however, the question of the cost of the grinding process, per pound of product, is scarcely significant in relation to the high value of these materials but the avoidance of contamination is a major problem. Thus, the inclusion of iron, from the balls and the mill body, in "face powder" is highly undesirable since such iron imparts a brown tint to the powder, and for this reason cosmetic and similar materials are usually ground in a mill having a ceramic body and with smooth natural pebbles as the grinding medium; that is, in a pebble mill. Similarly, pebble mills are used for grinding of feldspar for high-grade porcelain, since, as the previous case, minute traces of iron will introduce objectionable tinting. In this connection, it is of interest to note that there appears to be extant a licence for the removal of five tons of pebbles from Chesil Beach per annum; these pebbles, which are naturally graded for size, being ideal for use in pebble mills.

As an extreme example of the importance of the elimination of contamination may be quoted the grinding of minute quantities of metallic halides, used in certain classes of infra-red absorption spectroscopy, for which purpose a small vibration mill with body and balls made of agate is often used. In cases such as these, the decision as to wet or dry grinding is rarely based on economic considerations, but is usually made on the basis of a study of a number of extraneous factors, such as some special property of the material being processed, some desired condition of the final product or of the initial and final states of the materials involved.

Thus pyrotechnic materials might well be ground dry, since this would, in general, give a product having superior characteristics to one which has been ground wet, but if the material were very reactive it would probably be ground in an inert atmosphere. Similarly, tungsten carbide for use in the preparation of sintered carbide tool-tips would be ground dry, since moisture would complicate the subsequent processes and be harmful to the final product. Metallic powder pigments would probably be ground dry, since by this means discoloration is minimized. The grinding of aluminium for the preparation of paint is, however, often carried out wet since by this means stearic acid, or some other antiflocculent, may be introduced.

Finally, clays for the preparation of ceramic slips would reasonably be expected to be ground wet, since the initial condition of the clay is wet and it is required in a wet form for subsequent processing. Clearly, in such a case, dry grinding, which would involve drying, would be uneconomic and would only be adopted in very special circumstances. An application of the ball mill which can scarcely be classed as milling but which should, nevertheless, be mentioned is the dispersion of pigment in the varnish vehicle in connection with the manufacture of paint. The concentration of solid material in the suspension is, in this application, usually quite low, generally being about 10% by volume, and, although no appreciable reduction of the size of the particles occurs, considerable ball wear is often encountered. These observations are of some interest in view of a theory of the internal dynamics of the ball mill, propounded by one of the present writers, since that theory suggests that when the quantity of solid material is small in relation to the volume of the ball charge, the particle size reduction might be small but that the ball wear would be appreciable.

From the foregoing outline of the problems involved in the selection of a mill for a given purpose and in the operation of mills, it is apparent that the subject is very complex. Furthermore, much of the theory underlying a rigid solution of any problem is in a very fluid state and, in consequence, the solution of a practical problem generally involves the use of considerable judgement, based on previous experience, to temper the conclusions drawn from theoretical analysis.

CHAPTER 2

# THE MOTION OF THE CHARGE WITHIN A MILL

The motion of the charge, that is the grinding media and the material undergoing grinding, within a mill is of considerable theoretical interest and practical importance, and, for these reasons, has been the subject of considerable study by a number of workers, but, even so, no rigid and complete theory, covering all the aspects of the dynamics of the mill charge, has yet been produced. The practical importance of this subject clearly resides in the possibility of the prediction of the grinding behaviour, and other such characteristics, of a mill from the knowledge of the trajectories of the elements of the mill charge. The theoretical interest lies in the study of the dynamics of the system and in the derivation of equations to define the motion of the elements of the mill charge in terms of fundamental quantities such as the size and the speed of rotation of the mill. A simple example of the practical importance of this information is in the use of the knowledge of the trajectories followed by the balls in a mill to determine the speed at which the mill must run in order that the descending balls shall fall on the toe of the charge, and not upon the mill liner. The impact of the balls upon the liner plates can lead to unduly rapid wear of the latter, and so to high maintenance costs.

In this chapter the motion of the particles constituting the charge of the mill will be considered; it being assumed that there is no slip between the mill shell and the charge. There exists another type of motion, in which the charge as a whole slips relative to the shell, which gives rise to the phenomenon of "surging", but consideration of this type of motion will be deferred until a later chapter.

A study, from first principles, of the behaviour of a mill charge is much simplified if the charge is imagined to be composed of rods, instead of balls, since by this means the complication of any axial motion of the balls is eliminated and the problem is reduced to one in two dimensions. Consider first the motion of a single rod, of diameter $d$, within a smooth shell of internal diameter $D$; when the shell rotates about a horizontal axis with an angular velocity $\eta$ radians per second. In such a case the rod will lie near the lowest point of the mill, as in Fig. 2.1 and will rotate at such speed that the peripheral speed of the rod is the same as that of the shell. Furthermore, the displacement of the radius vector joining the centre of the mill and that of the rod would be such that the work done by reason of the couple formed by this displacement is equal to the energy dissipated in the distortion of the rod and shell at the line of contact.

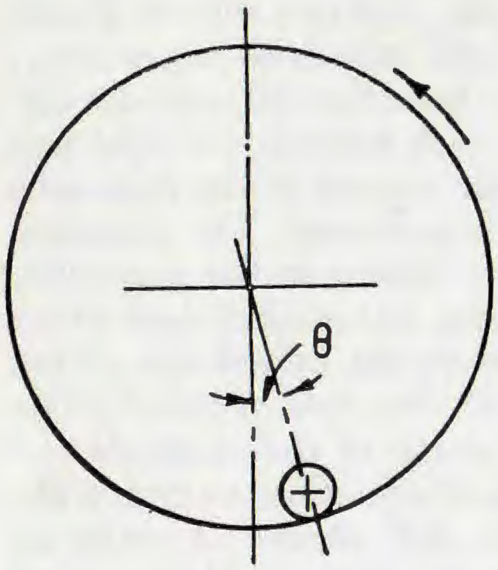


FIG. 2.1

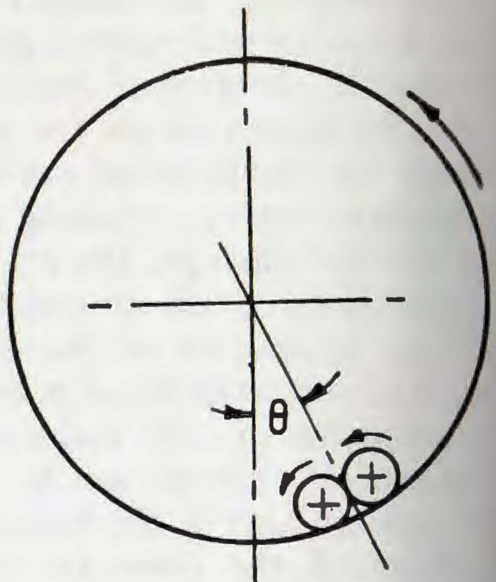


FIG. 2.2

If now two rods are placed within the mill, as in Fig. 2.2, the motions of the surfaces of the rods at the point of contact are in opposite senses and free motion inside the shell is eliminated. In this case the angle between the vertical and the radius vector joining the centre of the mill and the centre of gravity of the pair of rods is much greater than that for a single rod. But, again, equality exists between the work done to rotate the shell and that dissipated in friction at the contact points and in distortion of the metal surfaces. A further increase in the number of rods would enhance this effect until relative rotation between the rods is largely precluded and, in this respect, the charge behaves almost as a solid body.

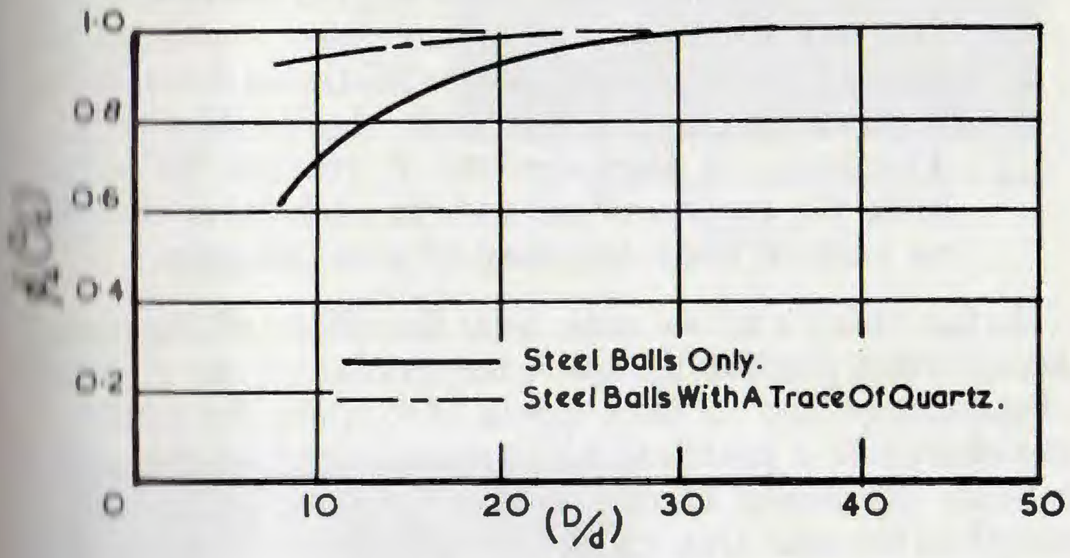


Fig. 2.3

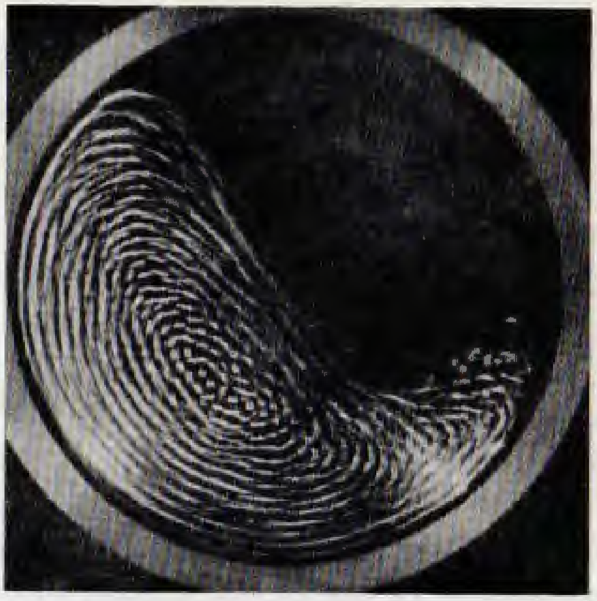
The correctness of this view is supported by the work of one of the present authors who, with a co-worker, Rose and Evans,(1) has shown that, for all other factors remaining equal, the power to drive a mill increases with the value of the ratio $(D/d)$; that is for increasing numbers of small rods (in the experiments balls were used). This relationship, shown in Fig. 2.3 indicates, at once, that the displacement of the centre of gravity of the charge increases with an increasing number of balls, which is in accordance with the reasoning given above. This figure also shows that the variation in the displacement of the centre of gravity is much less when the coefficient of friction between the surfaces is high, which indicates that, for this condition, the locking of the charge is effective when but a few elements are involved.

If the speed of rotation of the mill is so low that the effects of centripetal acceleration may be neglected, then the displacement of the centre of gravity of the charge will increase until either of two possible limiting conditions is reached; these conditions being:

(1) The tangential force at the shell, necessary to maintain the displacement of the mass centre of the charge, becomes equal to the frictional force. When this state is reached the whole charge slips back.

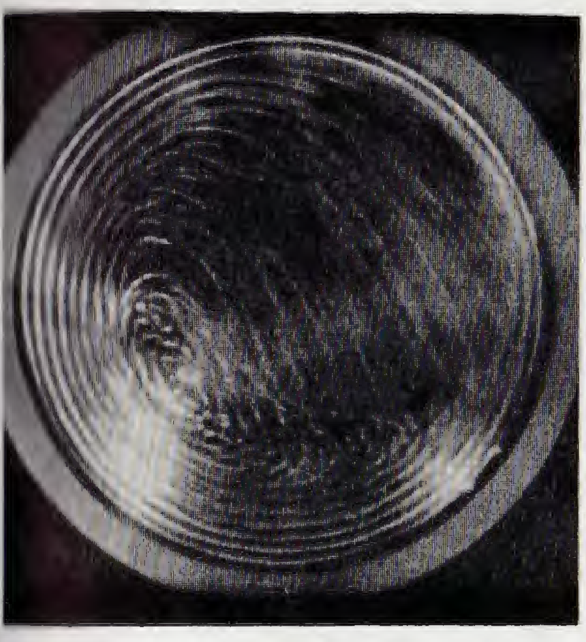
(2) The angle of displacement, $\theta$, reaches 30° when, since the centres of the rods in each layer then fall on vertical lines, the heap of rods collapses.

In fact this is a trivial case, since the effects of centripetal acceleration profoundly affect the motion of the elements of the charge, but for slow speeds of rotation the motion of the charge in a practical mill approximates to that given by case (2) above. In this motion the balls (or rods) will travel on circular arcs, concentric with the shell of the mill, until the point of instability is reached, after which they roll down the surface, which is inclined at about 30° to the horizontal, in a series of parallel layers. This motion is shown on a photograph taken through the transparent end of a model mill in Fig. 2.4. It will be noticed that a small "vortex" exists towards the middle of the charge. At higher speeds of rotation the balls no longer roll down the surface of the charge but, at a certain point, are projected into space and thereafter describe *approximately* parabolic paths before again meeting the ball mass; these ball paths being as shown in Fig. 2.5. There appear to be no universally adopted names for these two types of motion of the charge, but the evidence appears to be in favour of "cascading" for the first type and "cataracting"



*(Dr. J. M. Connor and the University of London)*

FIG. 2.4



*(Dr. J. M. Connor and the University of London)*

FIG. 2.5

for the second type. These names will be adopted for the present work.

As the speed of rotation of the mill is increased the particles are projected with progressively greater velocities until the theoretical trajectory for a particle, which is in fact lying against the mill shell, would fall wholly outside the shell. Clearly, since the particle cannot pass through the shell, it would lie against the shell throughout the cycle and so be carried around continuously with the mill shell. This condition is known as centrifuging and the speed of rotation at which it occurs, for the outermost layer of particles, is known as the "critical speed" of the mill. This speed, the critical speed, is of considerable importance in mill technology since, for example, other factors being equal, the equal performance of two mills of different sizes demands that the ratio of the actual running speed to the critical speed should be the same for the two mills. An expression for the critical speed of a mill will now be derived.

By reference to Fig. 2.6, it is easily seen that if the trajectory is *not* to fall *inside* the shell, the radius of curvature of the path, $\varrho$, must be greater than $R$, that is than that of the mill shell. Furthermore, this must be true for from wherever the trajectory might start.

From the dynamics of a particle it follows that for the trajectory,

$$\left.\begin{array}{ll} x' = V\cos\alpha & x'' = 0 \\ y' = V\sin\alpha - gt & y'' = g \end{array}\right\} \qquad \ldots. \quad (2.1)$$

Also, from elementary mathematics, the radius of curvature, $\varrho$, of any curve is given by

$$\varrho = \frac{\{1+(\mathrm{d}y/\mathrm{d}x)^2\}^{3/2}}{\mathrm{d}^2y/\mathrm{d}x^2}$$

which by substitution (for example, see Todhunter's *Differential Calculus*, p. 174) gives

$$\varrho = \frac{(y'^2+x'^2)^{3/2}}{x'y''-y'x''} \qquad \ldots. \quad (2.2)$$

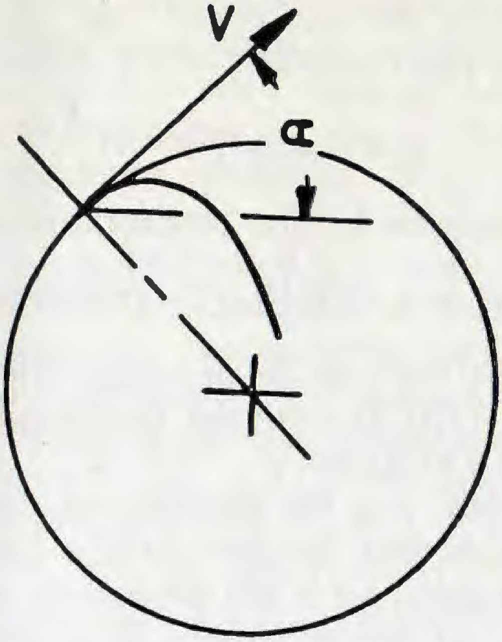


FIG. 2.6

Thus from the conditions specified $\varrho \geqslant R$, so, substituting from (2.1) into (2.2).

$$-R \leqslant \frac{V^3}{gV \cos \alpha}$$

$$\leqslant \frac{V^2}{g\cos \alpha}$$

or

$$\cos \alpha \geqslant \frac{V^2}{gR} \qquad \ldots. \; (2.3)$$

but $\cos \alpha \ngtr 1{\cdot}0$ so the limiting condition is $\alpha = 0$ which corresponds to

$$V^2/gR = 1$$

or, since $V = \omega R$, $\omega^2 = g/R$

from which $\omega = \sqrt{(g/R)} \qquad \ldots. \; (2.4)$

When the diameter of the ball is taken into consideration, the radius of the outermost ball path is given by $\frac{1}{2}(D-d)$ which, when substituted for $R$ in equation (2.4), gives

$$\omega = \sqrt{\{2g/(D-d)\}}$$

and this on expansion by the binomial theorem leads to

$$\omega = \sqrt{(2g/D)}\,.\,(1+0{\cdot}5d/D) \qquad \ldots . \quad (2.5)$$

If, as is usually the case in practice, the value of $d/D$ is less than about 1/20, the second bracketed term may, in general, be taken as unity.

In this treatment $\omega$ is the angular velocity of precession of the mill charge and, because of slip between the mill shell and the charge, this is not necessarily equal to $\eta$, the angular velocity of the mill shell. If, as is often the case, this slip is assumed to be negligible, then $\omega = \eta$ and these equations may be cast into the slightly more convenient practical form

$$\begin{aligned} N_c &= (60/2\pi)\sqrt{(2g/D)} \\ &= 76{\cdot}6\sqrt{(1/D)} \qquad \ldots . \quad (2.6) \end{aligned}$$

where $D$ is the mill diameter in feet and $N_c$ is the critical speed of the mill in r.p.m.

The above formula is based on the assumption that there is no slip between the ball charge and mill shell and, to allow a margin of error, it has been common practice to increase the coefficient in the equation by as much as 20%.

That relative slip between the charge and shell is of importance in this connection is supported by the work of Rose and Evans(1) who report that, using ground steel balls in a small-scale model mill made of solid-drawn brass tube, centrifuging did not occur even at running speeds of 120% of the critical value. However, when the same mill was fitted with effective lifters centrifuging occurred when the actual speed exceeded the theoretical value by a few per cent. This observation is confirmed by Gründer,(3) who showed that centrifuging in a mill with porcelain balls and body occurred when the actual speed exceeded the calculated critical value by about 5 % to 10 %. It is questionable, however, whether with modern liners maintained in reasonable condition the increasing of the value of the coefficient of equation (2.6) by 20 % is necessary or desirable.

Attention will now be turned to a more detailed analysis of the motion of the ball charge. In the first place, on the grounds of simplicity, the analysis first developed by Davis(4) will be considered, even though, as shown later, it is, in certain respects, an over-simplification.

Consider a point P, Fig. 2.7, at which the projection of the particle occurs and let this point be at a distance $r$ from the centre of the mill. In this connection, the "point of projection" may be considered to be the point at which the trajectory of the particle is influenced by gravity and is not wholly controlled by the packing of the surrounding charge.

For projection to occur, it is necessary that the radius of curvature of the path under the influence of gravity should be smaller than the radius of curvature of the initial circular path. An expression for the radius of curvature of the path under gravity is given by equations (2.2) and (2.3) from which

$$r \leqslant V^2/(g \cos \alpha)$$

which, for the limiting case and since $V = \omega r$,

$$\cos \alpha = \omega^2 r/g \qquad \ldots . \ (2.7)$$

From the geometry, however, it is clear that the line of projection PA must be at right angles to the radius vector OP so $\beta = \alpha$ and hence

$$r = (g/\omega^2) \cos \beta \qquad \ldots . \ (2.8)$$

This equation is satisfied when point A is distance $g/\omega^2$ above the centre of the mill, and furthermore, since the

$V = \omega^2 r$ ? .

or $V = \omega r$

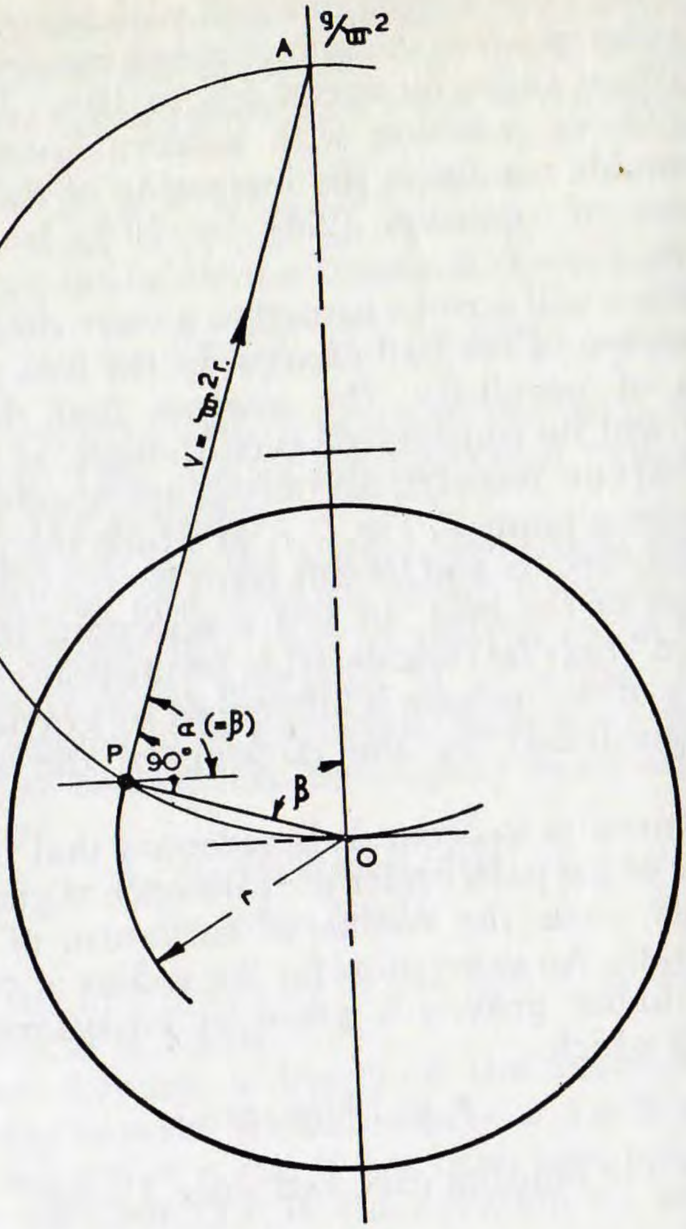


FIG. 2.7

angle OPA is a right angle and from the theorem of geometry that the angle within a circle is also a right angle, it follows that the line from which projection takes place is an arc of a circle of radius $g/2\omega^2$; the "Davis circle".

By a simple substitution this leads to the expression

$$r_c = 0{\cdot}408/n^2 \qquad \ldots . \;(2.9)$$

where $r_c$ is the radius of the Davis circle and $n$ is in r.p.m. Again, in this treatment, slip is usually neglected and so $n$ represents the speed of the mill in r.p.m.

The trajectories of the ball paths may, on the assumption that the ball paths are parabolic, be easily plotted. Thus, by the integration of equations (2.1) and taking the origin of the co-ordinate system at the point of projection.

$$\left.\begin{aligned} x &= Vt\cos\alpha \\ y &= Vt\sin\alpha - \tfrac{1}{2}gt^2 \end{aligned}\right\} \quad \ldots\ldots (2.10)$$

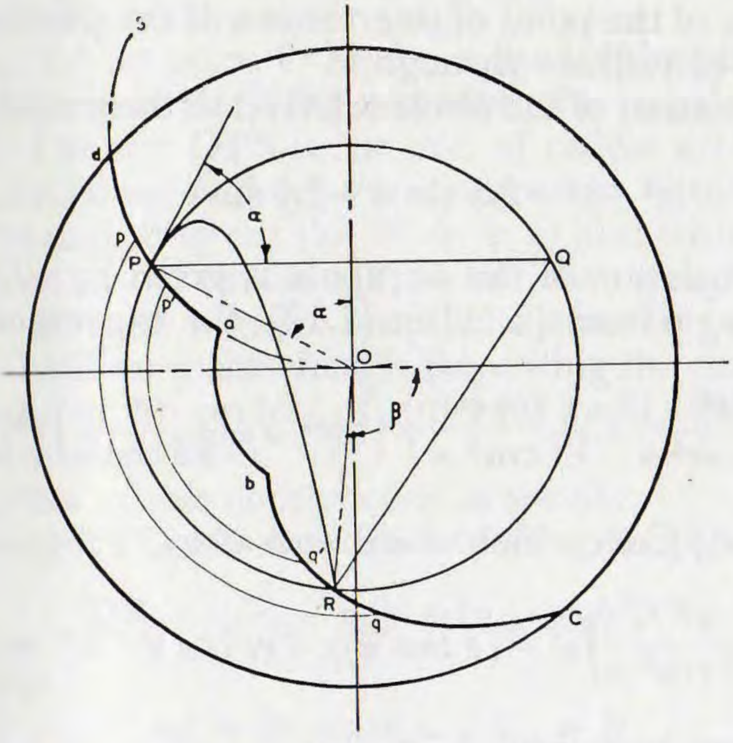


FIG. 2.8

and eliminating $t$ between these equations

$$y = x\tan\alpha - \frac{gx^2}{2V^2\cos^2\alpha} \quad \ldots\ldots (2.11)$$

from which the parabolic trajectories may be plotted.

More important than a knowledge of the form of the trajectories is a knowledge of the form of the surface in which the parabolic trajectories terminate.

From Fig. 2.8, it is clear, from requirements of continuity, that the amount of material entering any annular element p′p q′q must be equal to the quantity leaving it and so, since the parabolic trajectories cannot cross, a trajectory must terminate in the same annular element as it originates. Thus, the surface bRc is defined by points, such as R, which are obtained by the intersection of the parabolic trajectory with a circle of radius $r$; where $r$ is the radius at which the trajectory originates. A representative point, such as R, may be obtained by the determination of the values of $x$ and $y$, taking the point of projection P as the origin, of the point of intersection of the parabola with the circle of radius $r$ through P.

The equation of the circle referred to these co-ordinates is

$$x^2+y^2+2rx \sin \alpha-2ry \sin \alpha = 0 \quad \ldots . \quad (2.12)$$

and the equation of the parabola is given by (2.11). On substituting $y$ from (2.11) in (2.12), the expression

$$\frac{g^2x^4}{4V^2 \cos^4 \alpha}-\frac{g \sin \alpha}{V^2 \cos^3 \alpha}+\left(\sec^2 \alpha-\frac{rg}{V^2 \cos \alpha}\right)x^2 = 0$$

is obtained; from which $x^2=0$ and also

$$\left(\frac{g^2}{4V^2 \cos^2 \alpha}\right)x^2-(g \tan \alpha)x-gr \cos \alpha+V^2 = 0$$

Now from equation (2.7),

$$V^2 = gr \cos \alpha$$

which substituted in the immediately preceding equation, leads to

$$\frac{x^2}{4r \cos^3 \alpha}-x \tan \alpha = 0$$

The solutions of this equation are $x = 0$

and $$x = 4r \sin \alpha \cos^2 \alpha \quad \ldots . \quad (2.13)$$

It is clear that, in general, a parabola cuts a circle in four points and, in the present case, three roots of the quartic equation are zero whilst the fourth, given by equation (2.13), is that required.

When this value for $x$ is substituted in the equation for the parabola, it is found that,

$$y = -4r \sin^2 \alpha \cos \alpha \qquad \ldots . \ (2.14)$$

Since, for most purposes it is not necessary to plot the whole trajectory, the representative point R may be obtained geometrically thus:

In Fig. 2.8 let point O be the centre of the mill and let the concentric circle RPQ represent the circular path of the ball. The arc OPS is the arc, of radius $g/2\omega^2$, representing the line at which projection occurs. Project a horizontal through P to cut the circle at Q and with centre Q describe an arc of radius QP, to cut the circle at R. The point R is then a point on the surface in which the "flying" balls meet the charge. Repeating the construction for a number of circles, of different radii, leads to the required surface bRC.

The proof of this construction is simple:

The length $PQ = 2r \sin \alpha$, and the length QR is given by

$$QR = \{(x_Q - x_R)^2 + (y_Q - y_R)^2\}^{1/2}$$

However,

$$x_Q = 2r \sin \alpha; \qquad y_Q = 0$$

so

$$x_R = 4r \sin \alpha \cos^2 \alpha; \qquad y_R = -4r \sin^2 \alpha \cos \alpha$$

or

$$QR = \{(2r \sin \alpha + 4r \sin \alpha \cos^2 \alpha)^2 + (4r \sin^2 \alpha \cos \alpha)^2\}^{1/2}$$

which on expansion and simplification leads to

$$QR = 2r \sin \alpha$$

Thus, $PQ = QR$ and this is precisely the relationship used in the construction.

The curve ab is a circular arc struck from the centre of the mill and represents a surface which does not osculate with the segment of circle OPS, and, in consequence, from which no projection takes place. The radius of this arc is fixed by the magnitude of the mill charge and, denoting the radius by $r_i$, Davis gives the expressions

$$r_i/R = \text{K} \qquad \ldots. \; (2.15)$$

where

$$K = -0{\cdot}024 + 0{\cdot}39\sqrt{(7-10J)} \qquad \ldots. \; (2.16$$

approximately; $J$ being, in this expression, the fractional filling of the mill.

Since $K$ cannot be less than zero, it follows that the mill filling cannot exceed about 0·7 without interference between the particles taking place. Actually Davis states that, in order that interference shall not occur, $r_i$ should not be less than $0{\cdot}228/n^2$, where $n$ is the speed in revolutions per *second.*

The fraction of the charge which is in "flight" can be calculated fairly simply thus.

The angle $\beta$ in Fig. 2.8 is given, from equations (2.14), by

$$\begin{aligned} \sin\beta &= (1/r)(4r\sin^2\alpha\cos\alpha - r\cos\alpha) \\ &= (4\cos^3\alpha - 3\cos\alpha) \end{aligned}$$

From various trigonometrical identitites this leads to

$$\sin\beta = \cos 3\alpha, \quad \text{but} \quad \cos 3\alpha = \cos(180-3\alpha)$$

and so, since $\sin\beta = \cos(90-\beta)$,

$$\cos(90-\beta) = \cos(180-3\alpha)$$

or

$$90-\beta = 180-3\alpha$$

or

$$-\beta = 90-3\alpha \qquad \ldots. \; (2.17)$$

Now, the angle passed through by the particle whilst in the parabolic path is $\alpha+90+\beta$ which is $\alpha+90+3\alpha-90$ or $4\alpha$.

If the speed of the mill is $n$ r.p.m., the time per revolution, $T_r$, is $1/n$ and the time, $T_c$, for the ball to cover the circular path is given by

$$T_c = T\left(\frac{360-4\alpha}{360}\right)$$

Also, the time required to traverse the parabolic path is given by

$$\begin{aligned} T_p &= \frac{x}{V\cos\alpha} \\ &= \frac{2\sin\alpha\cos\alpha}{\pi n} \qquad \ldots. \quad (2.18) \end{aligned}$$

If now it is assumed that the motion of the charge may be represented by the motion of the particle at the radius of gyration of the charge, it is necessary to find the angle α corresponding to the radius

$$r = \sqrt{\frac{R^2+r_i^2}{2}}$$

and this angle will be found to be 54° 44′.

By use of this angle and the above expressions, it is found that

$$T_r = 1/n; \qquad T_c = 0{\cdot}39/n; \qquad T_p = 0{\cdot}3/n$$

Thus, the time spent in the circular path, as a fraction of the time of the cycle is

$$\frac{0{\cdot}39}{1-0{\cdot}3} = 0{\cdot}56 = 56\,\% \qquad \ldots. \quad (2.19)$$

Also, the number of cycles per revolution is given by

$$T_r/T = 1/0{\cdot}7 = 1{\cdot}45 \qquad \ldots. \quad (2.20)$$

Thus, it follows that, as a first approximation, 56 % of the time is spent in the circular path and also that the area between the mill shell and the curves dPabRC is 56% of the area occupied by the charge when the mill is at rest. It also follows that, on an average, every ball strikes another 1·45 times per revolution of the mill.

The analysis of Davis, just outlined, is open to criticism on a number of grounds. For example, the frictional characteristics of the charge are ignored, the effects of interactions between the elements of the charge in a given trajectory and the interference between adjacent trajectories are neglected, as are the centripetal forces acting on the particles. Furthermore, some of the equations involved appear to include a number of rather wide approximations. Even so this work was the first serious attempt to derive a rational theory of the ball mill and, whilst forming a good basis for mill calculations, it had also paved the way for later studies.

The first important modification to the above treatment was suggested by von Steiger(5); who pointed out that the hypothesis of free flight of the particle along the ascending branch of the parabolic trajectory is invalid since the continuous projection of material along any trajectory results in contact between the particles. Thus for the ascending branch of the parabola the speed along the arc is constant; not variable as is the case with free flight of particles.

On referring to Fig. 2.9, it is seen that the intrinsic equation to the normal $\mu$ is, since the radial acceleration is provided by the normal component of the gravitational acceleration, given by

$$V^2/\varrho = g\cos\alpha$$

where $\varrho$ is the radius of curvature of the trajectory at the point P.

Thus

$$-\frac{V^2}{\mathrm{d}s/\mathrm{d}\alpha} = g\cos\alpha \qquad \ldots. \; (2.21)$$

or

$$\frac{V^2}{g}\,\mathrm{d}\alpha = -\cos\alpha\,\mathrm{d}s = -\mathrm{d}x$$

from which

$$\frac{V^2}{g}(\alpha_0-\alpha) = x-x_0 \qquad \ldots. \; (2.22)$$

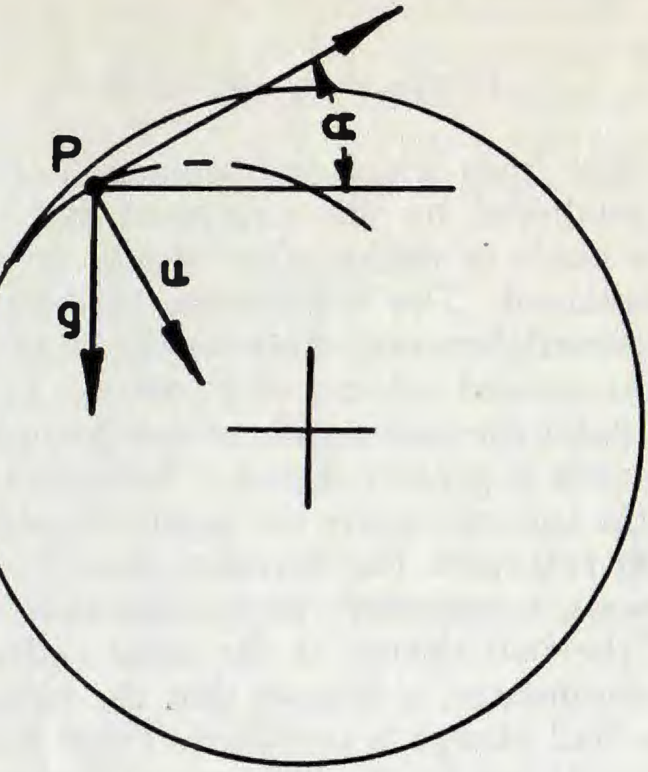


FIG. 2.9

Also from equation (2.21),

$$\frac{V^2}{g}\frac{d\alpha}{\cos\alpha} = -ds$$

but since $dy = ds \sin\alpha$,

$$dy = -\frac{V^2}{g}\frac{\sin\alpha\, d\alpha}{\cos\alpha}$$

from which

$$y - y_0 = \frac{V^2}{g}\Big[\log\cos\alpha\Big]_{\alpha_0}^{\alpha}$$

$$= \frac{V^2}{g}\log\frac{\cos\alpha}{\cos\alpha_0} \qquad \ldots\ldots \quad (2.23)$$

For summit $x = x_s$ and $\alpha_s = 0$, so from equation (2.22),

$$x_s - x_0 = \frac{V^2}{g}\alpha_0 \qquad \ldots\ldots \quad (2.24)$$

and from equation (2.23),

$$y_s - y_0 = \frac{V^2}{g} \log \sec \alpha_0 \qquad \ldots . \; (2.25)$$

The surface from which the projection of the particles occurs is unaltered by this treatment and is, as before, given by a circle of radius $g/2\omega^2$ drawn in the way previously explained. The trajectories themselves are considerably altered, however, since for the same initial conditions, the horizontal velocity of a particle, at the summit, is greater under the assumption of von Steiger, and so the particle travels a greater distance horizontally. Also the height of the summit above the point of projection is different in the two cases. Furthermore, since from continuity considerations, a trajectory on termination must cut the surface of the ball charge at the same radius as that at which it commences, it follows that the equilibrium surface of the ball charge is modified. These differences are clearly brought out by the diagram of Fig. 2.10 in which trajectories starting at the same point are plotted in accordance with the two treatments and the corresponding surfaces of the charge are shown.

As already stated, the motion of a mill charge is considerably more complicated than the previous treatments would imply, and so far the various proposed treatments are not completely satisfactory.

One of the most comprehensive analyses, and that upon which most of the remainder of this chapter is based, is that developed by Barth(6) and, independently of Barth, by Hinsley and Fobelets.(7) *

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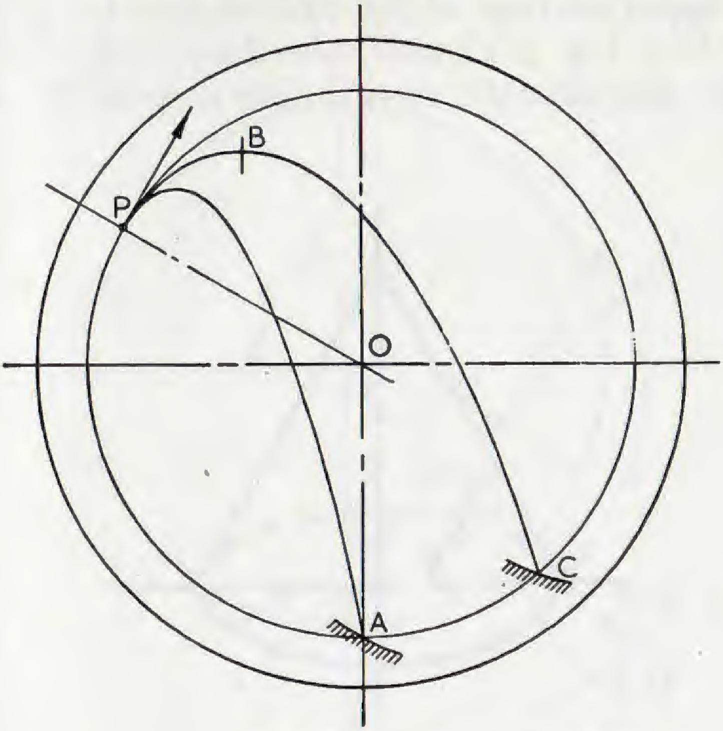
* This work, which has not previously been published, was developed over a number of years by Messrs. Hinsley and Fobelets, who were apparently unaware of the prior work of Barth.

Messrs. Hinsley and Fobelets have very kindly made all their material available to the authors and much of this chapter was written, on the basis of their material, before the work of Barth came to the notice of the present writers.

The present authors are pleased to acknowledge their indebtedness to Messrs. Hinsley and Fobelets in this matter.

In the first place the equilibrium surface of the mill charge is determined and for this purpose the effective coefficient of friction* of the charge (balls plus powder) must be established. A generalized coefficient of friction may be determined by forming a large conical heap of the mill charge on a flat surface, as shown in Fig. 2.11; the heap being formed by shovelling the material on to the



Curve PA — Parabolic Trajectory.
PB — Trajectory Of Von Steicer.
BC — Parabolic Trajectory.

Fig. 2.10

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* In this overall coefficient there are components such as the coefficient of friction between the ball and the surface of the powder particle; a "ploughing" factor representing the force required to move the tip of the ball through the surface layers of the powder particle in question, and a factor which, for want of a better expression, can be called a mechanical displacement factor, the magnitude of which is determined largely by the mode of packing of the particles and the coefficient of friction between the particles which are rubbed against one another during displacement. The magnitude of these components is determined by physical characteristics of the powder material; for example, by the hardness, which will determine the degree of ploughing of point of contact of the ball through the surface of a particle.

apex of the cone and allowing the stable angle of repose to become established. The angle of repose, $\gamma$, is then measured and the coefficient of friction $\lambda$ is given by

$$\lambda = \tan \gamma \quad \ldots . \quad (2.26)$$

Let the upper surface of the mill charge be given by the curve A–B of Fig. 2.12 and take the centre of the mill as the origin; also take the co-ordinate axes as shown. Let the mill shell rotate with an angular velocity $\eta$ and also assume that the particles of the charge describe circular arcs around the origin with this velocity.

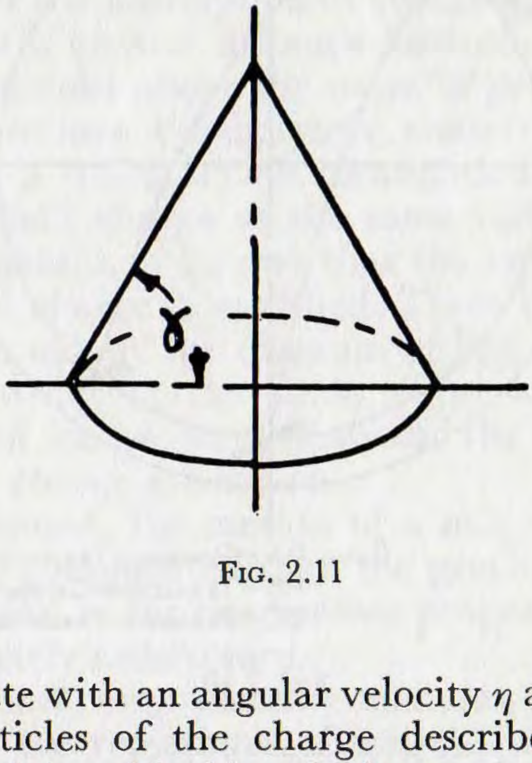

FIG. 2.11

Now any particle, P, of mass $m$, on this surface, at a distance $r$ from the origin, will be in equilibrium under the influence of three forces:

(1) The radial "centripetal" force: $mr\eta^2$

(2) The weight: $mg$

(3) Frictional forces acting between the particle and its neighbours.

On equating the forces acting on the particle the following expression is obtained:

$$\lambda mr\eta^2 \cos(\beta-\alpha)+\lambda mg \sin\beta + mr\eta^2 \sin(\beta-\alpha) = mg\cos\beta \quad \ldots . \quad (2.27)$$

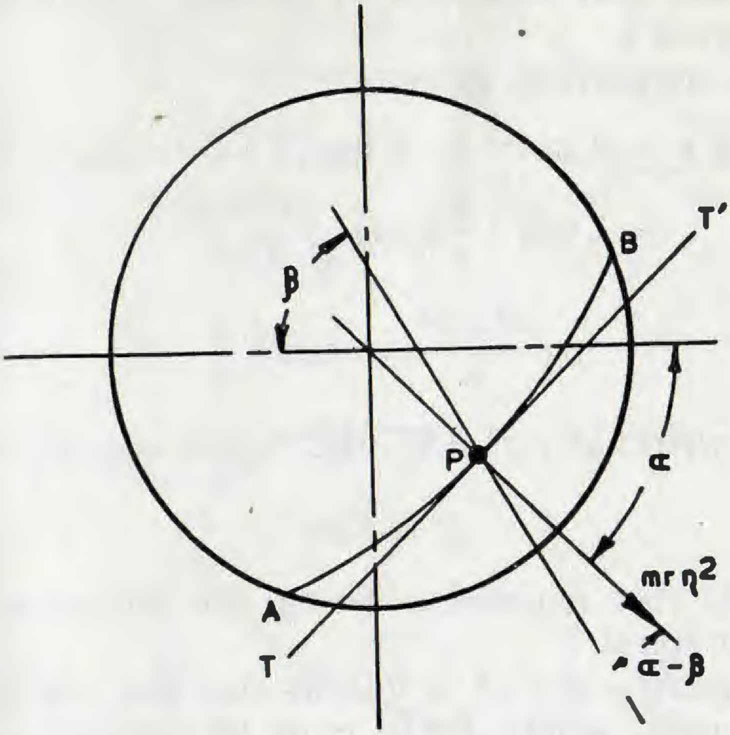

FIG. 2.12

Multiply this equation by $1/m\eta^2$ and rearrange.

$$\lambda(g/\eta^2 \sin\beta + r\cos\beta\cos\alpha + r\sin\beta\sin\alpha) = g/\eta^2\cos\beta - r\sin\beta\cos\alpha + r\cos\beta\sin\alpha$$

and then substitute $x=r\cos\alpha$, $y=r\sin\alpha$, and also divide by $\sin\beta$ which then gives

$$\lambda(g/\eta^2 + x\cot\beta + y) = g/\eta^2\cot\beta - x + y\cot\beta$$

However, $\cot\beta = \mathrm{d}y/\mathrm{d}x$, so

$$\mathrm{d}y/\mathrm{d}x(g/\eta^2 + y - \lambda x) = -\lambda(g/\eta^2 + y) - x$$

Now let $z = (g/\eta^2 + y)$, which, on substitution, gives,

$$(z - \lambda x)(\mathrm{d}z/\mathrm{d}x) = -\lambda z - x$$

$$\frac{dz}{dx} = \frac{\lambda z + x}{\lambda x - z}$$

This is the standard form of homogeneous equation of first order and may be solved by putting $z = kx$, where $k$ is a function of $x$.

This, on integration, gives,

$$\log x = \lambda \tan^{-1} k - \tfrac{1}{2} \log (1 + k^2) + \log C$$

$$= \lambda \tan^{-1} \frac{z}{x} - \log \sqrt{\frac{x^2 + z^2}{Cx^2}}$$

or

$$\log \frac{x^2 + z^2}{c} = \tan^{-1} \frac{z}{x}$$

Now if we write: $x^2 + z^2 = R^2$, $\tan^{-1} (z/x) = \psi$, the expression

$$R = Ce^{\lambda\psi} \qquad \ldots . \quad (2.28)$$

is obtained; this expression being the equation of an equiangular spiral.

Also, since $R^2 = x^2 + z^2$, it follows that the origin of the spiral (the pole) where $R = 0$, must be situated at $x = 0$, $y = -g/\eta^2$; that is on the vertical axis at a distance $g/\eta^2$ above the centre of the mill. (It should be noted that this distance is also the diameter of the circle, the Davis circle, defining the surface at which projection occurs.) The coefficient $C$ is a parameter which is related, but not simply, to the degree of filling of the mill.

The configurations of the surface of the charge with various values of speed, coefficient of friction and mill filling, as calculated on this basis, are shown in Fig. 2.13. Examination of this figure shows that, for a given mill filling and coefficient of friction, the surface configuration of the charge is not greatly altered by variations in $(N/N_c)$ the ratio of the running speed to the critical speed. Similarly, for a given value of the ratio $N/N_c$ and a given

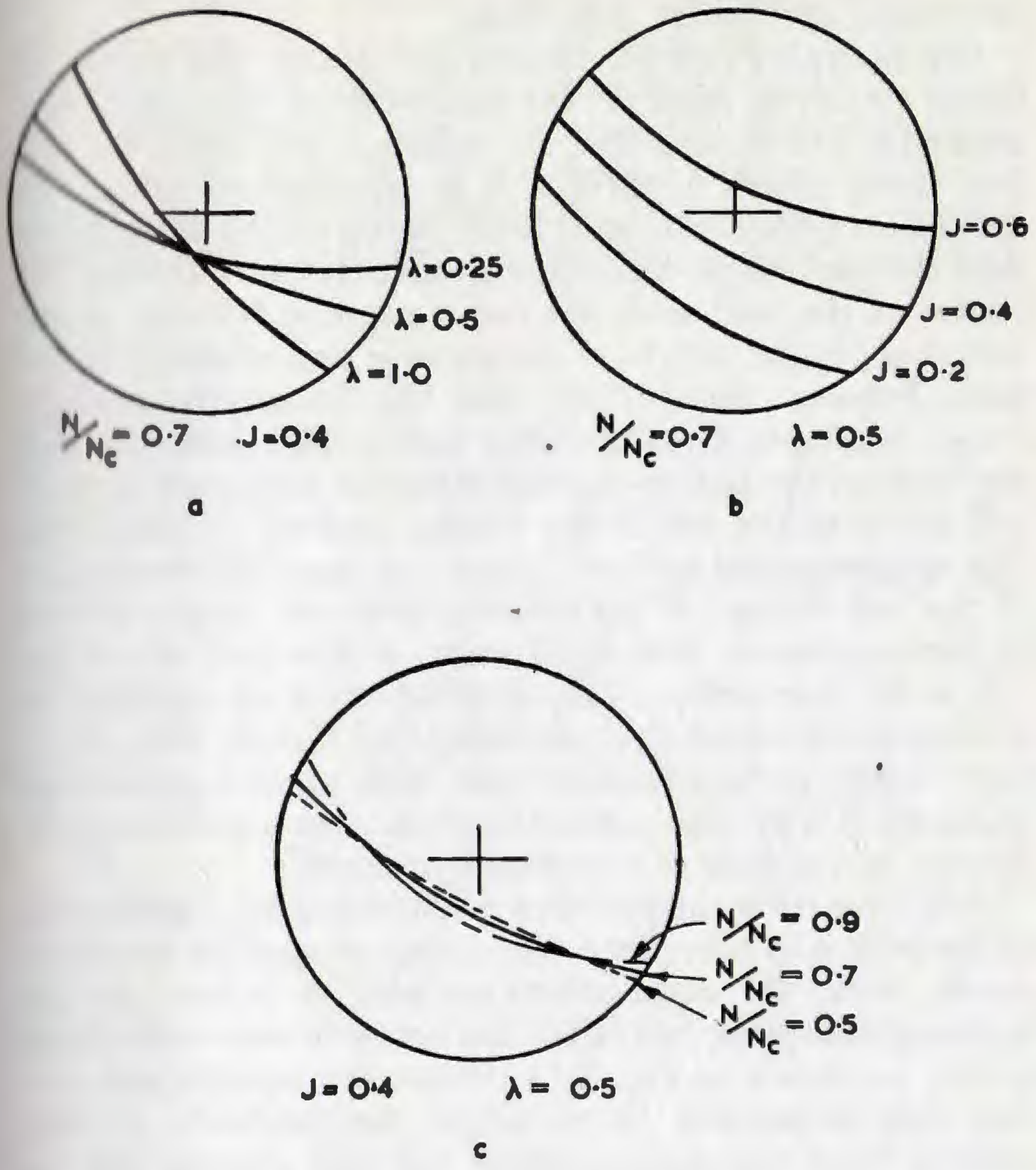

FIG. 2.13

coefficient of friction, the slope of the surface does not alter greatly with the mill filling; furthermore, the line defining the surface is almost straight. In fact the average slope of the surface increases very slightly with decreasing filling.

The variable which has the greatest influence on the configuration of the surface of the charge is clearly the coefficient of friction and, all other variables remaining unaltered, the slope of the surface increases rapidly with increasing coefficient of friction.

It is desirable that the meaning of this surface should be borne clearly in mind. If the derivation of the equation is studied it will be seen that the equation represents the surface upon which a particle is in equilibrium under the influence of the centripetal force, based on the assumption that the particle is traversing a circular path around the centre of the mill with the same angular velocity as the mill shell, of the weight of the particle and of the frictional force between the particle and the underlying surface. Now, clearly, in a real mill the balls which come up from the body of the ball mass must either be projected or must roll down to the toe of the charge and so, in cascading, this mathematical surface cannot represent the *free* surface of the ball charge. It is probable, however, that a surface of approximately this form exists a few ball diameters below the free surface; that is, there exists an equilibrium surface down which the cascading ball charge rolls. Thus, such a free surface cannot exist with cascading motion, although it is probable that one does, over a portion of the charge, in the case of cataracting motion.

It is clear from the previous discussion that, if projection of the balls is to take place, the charge must cross the Davis circle. Now, the equilibrium surface, as defined by the equiangular spiral, can fall either below or above the Davis circle; as shown in Fig. 2.14. When the equilibrium surface falls below the Davis circle, the particles, as they emerge from the main mass of the ball charge, pile up until the Davis circle is reached; as shown in Fig. 2.15. This is clearly possible since in the mass of balls lying above the main equilibrium surface but below the Davis circle, there exists an infinite number of equilibrium surfaces such as ab.

The inner bounding curve for this mass of balls is a circle concentric with the mill shell and tangent to the equilibrium surface. The position of this circular arc and the equilibrium surface is, of course, fixed by the condition that the "rolling" mass of balls and the "flying" mass are together equal to $J$, the static filling of the mill. Since, in this case, the projection takes place from the Davis circle, the construction for the termination of the trajectories, given earlier, is valid. In the second case, in which the equilibrium surface falls above the Davis circle, projection does not occur until the equilibrium surface is reached.

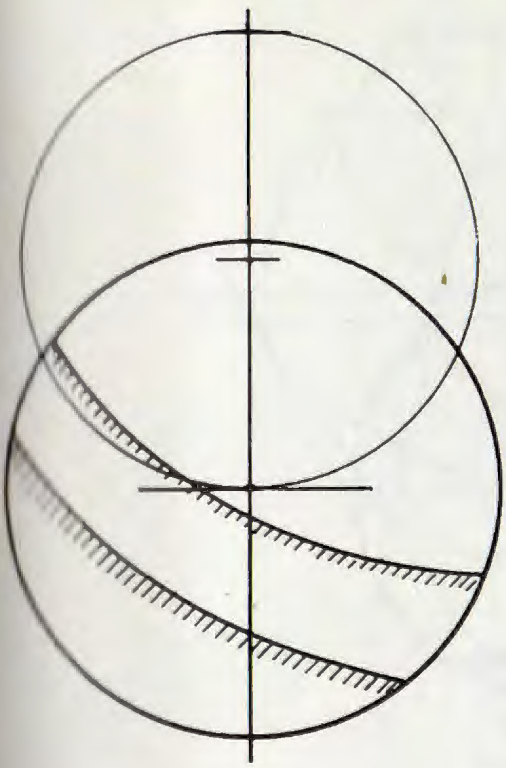

FIG. 2.14

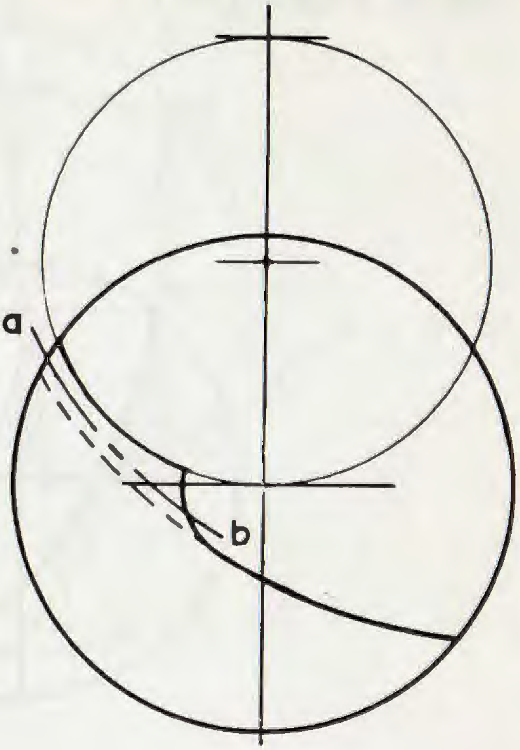

FIG. 2.15

Since projection does not now occur at the Davis circle, the previous construction must be modified.

By reasoning very similar to that used previously, it can be shown that the termination of a parabolic trajectory, commencing at a point P′ above the Davis circle, is obtained when, on Fig. 2.16, distance $P'Q' = Q'R$ *and the point* Q′ *is on the Davis circle.* Thus, the construction is similar to the previous except that point Q′ is on the Davis circle whereas point Q is on the mill circle. So for either case the point of termination of the trajectories may be determined by simple graphical methods.

It is interesting to have a knowledge of the proportion of the charge which is in "flight", and of that which is "rolling"; also to see how these quantities vary with the speed of rotation of the mill and with the coefficient of friction of the charge material. A simple construction for this purpose will now be described; this construction being due to Fobelets.(8)

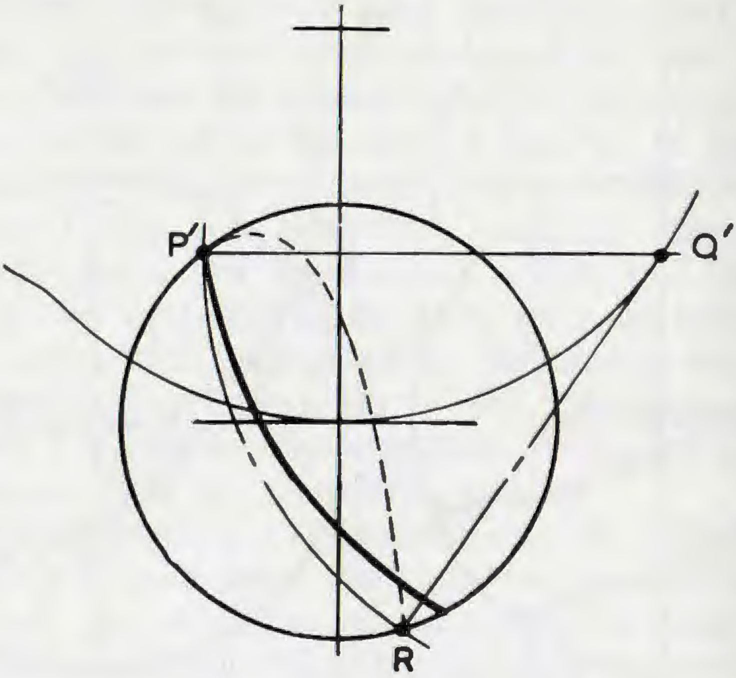

Fig. 2.16

Case 1. The point of projection on the Davis circle:

(a) As in Fig. 2.17, draw the mill shell circle and the Davis circle.
(b) With compasses determine position of point R; the termination of the trajectory originating at P. (Use point Q on mill circle as centre.)
(c) From the pole C (on the Davis circle), draw radius vectors at 10° intervals; taking vector CR as datum.
(d) From C set out along the successive rays a length $\overline{Ca} = \overline{CR} . e^{-\lambda(\pi/180)\delta}$, where $\overline{CR}$ is the length CR

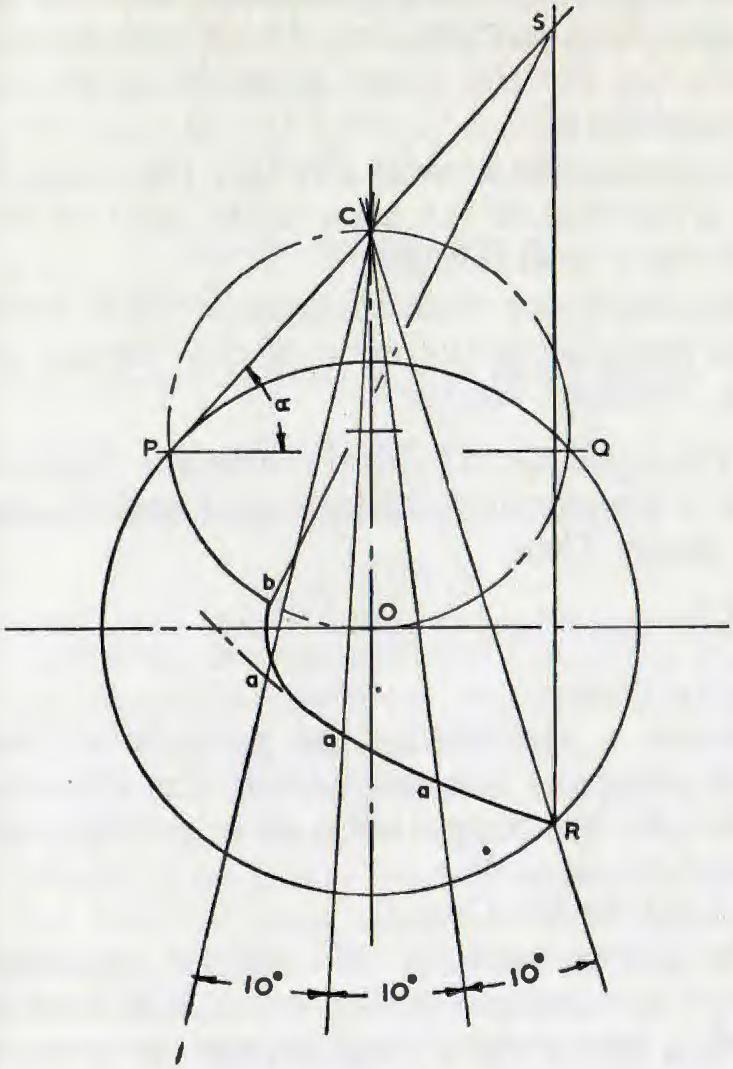

FIG. 2.17

measured from the figure, $\delta$ is the angle, in degrees, from the line CR to the ray under consideration and $\lambda$ is the coefficient of friction of the charge.

(e) Join the points such as "a" to give a smooth curve, from the centre of the mill, O, draw an arc of a circle tangential to this curve and to cut the Davis circle at b.

(f) Draw a line from P through C to intersect a vertical drawn through point R at point S.

(g) Repeat (b) and (f) using mill circles of various radii; and so establish the curve bS. (For most purposes this step may be dispensed with and points bS joined by a straight line. There is then some loss of accuracy but this is not generally of great practical importance.)

(h) Determine the area SbaRPCS. This when expressed as a fraction of the area of the mill circle gives $J$, the static mill filling.

(i) Determine the ratio of areas PCSbP and PbaRP; this ratio being the ratio of the "flying" charge to the "rolling" charge.

When the equiangular spiral enters the Davis circle the treatment is slightly more complicated and demands some trial and error. Thus,

CASE 2. The equiangular spiral enters the Davis circle:

(a) As for Case 1.

(b) Assume a position for the point of projection, P′, and using the construction of Fig. 2.16, determine point R; the termination of a parabolic trajectory which starts at P′.

(c) and (d) As for Case 1.

(e) Join points such as "a" *and the equiangular spiral should pass reasonably close to* P′. If it does not, start with a new point P′ and repeat the construction.

(f) to (i); As for Case 1.

The basis of this construction is as follows.

If the flight trajectories are parabolic, then the horizontal velocity $x'$ is constant and given by

$$x' = \omega r \cos \alpha \qquad \ldots . \ (2.29)$$

where $\alpha$ is the angle of projection and the other symbols have the same meaning as before.

Also the time of flight, $T_p$, is given by

$$\begin{aligned} T_p &= x/x' \\ &= x/\omega r \cos \alpha \qquad \ldots . \ (2.30) \end{aligned}$$

During this time, however, the rotation of the mill will bring, from the depth of the mass of the charge at radius $r$, a column of balls of length $\omega r T_p$.

This is the quantity of balls projected in time $T_p$, from a strip of width d$r$, and this may be written d$q_f$. Then

$$\begin{aligned} \mathrm{d}q_f &= \omega r(x/\omega r \cos \alpha) \\ &= x/\cos \alpha \qquad \ldots . \ (2.31) \end{aligned}$$

From Fig. 2.17, however, it is evident that

$$x/\cos \alpha = \overline{\mathrm{PS}} \qquad \ldots . \ (2.32)$$

Then, from equations (2.31) and (2.32)

$$q_f = \int \overline{\mathrm{PS}}\, \mathrm{d}r$$

where the integration is carried out along the curve from b to P. This integral is, however, equal to the area of the figure bPCS and hence the construction.

Although there is still an element of approximation in this treatment, it appears to be superior to that of Davis in that the effects of frictional characteristics of the charge are included and the computation of the "flying" charge does not involve reference to some mean radius. On the basis of this treatment it is possible to study the conditions for cataracting or cascading in some detail and this will now be done.

It will be noticed that, so far, no rigid definition of cascading and cataracting has been given. In fact such a definition is probably impossible since it now appears that there are not two types of motion of the charge involved, but a single type of motion in which, in certain cases, some details of the motion are not apparent. For the present purpose it is probably sufficient to name the motion "cataracting" when an appreciable gap exists between the parabolic trajectory of the innermost particle and the upper surface of the rolling charge. Cascading then corresponds to the motion in which this gap is so small as to give, in general, the appearance of being non-existent.

This difference is clearly shown in Fig. 2.18, which are the trajectories for a mill running with a definite charge at a definite speed but with different fillings. In Fig. 2.18a, with the higher filling, the space is very small and when allowance is made for the finite size of balls the mill would appear to be cascading. In the second illustration there is a definite space which, even allowing for interference between the balls in the different trajectories, would not be obliterated, and hence the mill would be described as cataracting. For the present purpose it will be assumed

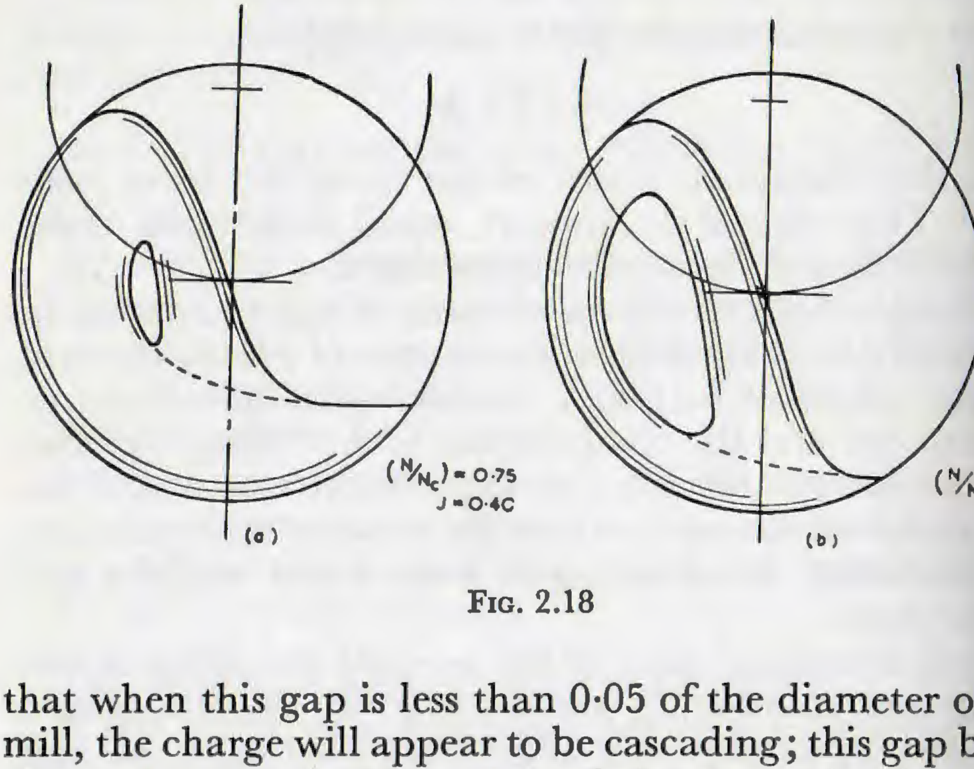

FIG. 2.18

that when this gap is less than 0·05 of the diameter of the mill, the charge will appear to be cascading; this gap being chosen because, in view of the finite size of the ball in a real mill, it is improbable that a gap smaller than this figure would be discerned. By the plotting of a number of such figures the conditions for a gap of less than the specified size to occur are soon established. The construction for the determination of the "flying" and "rolling" ball charges is then applied to each of these limiting cases when the results given in Table 2.1 are obtained. Since any *increase* in the mill filling *above* these values will reduce the size of gap it follows that these values are the *maximum* static mill filling for which *cataracting* will be apparent.

TABLE 2.1

Maximum static mill filling for cataracting to occur

| $\lambda$ | $\frac{N}{N_c}$ | Area of mill circle (in$^2$) | Area giving "rolling" charge (in$^2$) | Area giving "flying" charge (in$^2$) | $J_{\max}$ (%) | Ratio $\frac{M_f}{M_t}$ | Ratio $\frac{M_r}{M_t}$ |
|---|---|---|---|---|---|---|---|
| 0·5 | 0·5 | 3·14 | 0 | 0 | 0 | 0 | 1·0 |
| | 0·6 | 3·14 | 0·50 | 0·15 | 20·5 | 0·23 | 0·77 |
| | 0·75 | 3·14 | 0·81 | 0·50 | 41·0 | 0·38 | 0·62 |
| | 0·9 | 3·14 | 1·28 | 0·9 | 69·8 | 0·42 | 0·58 |
| 1·0 | 0·6 | 3·14 | 0·46 | 0·15 | 19·0 | 0·25 | 0·75 |
| | 0·75 | 3·14 | 0·90 | 0·5 | 45·0 | 0·36 | 0·64 |
| | 0·9 | 3·14 | 1·26 | 0·9 | 69·1 | 0·42 | 0·58 |
| 2·0 | 0·6 | 3·14 | 0·38 | 0·15 | 17·1 | 0·28 | 0·72 |
| | 0·75 | 3·14 | 0·82 | 0·50 | 41·5 | 0·38 | 0·62 |
| | 0·9 | 3·14 | 1·15 | 0·9 | 65·0 | 0·42 | 0·58 |

It is seen that a fourfold variation in the coefficient of friction produces, for a given speed of rotation, but a negligible variation in the maximum mill filling which will permit cataracting. Thus, it is probably safe to say that these results, shown graphically in Fig. 2.19, are applicable to any practical mill with an accuracy sufficient for any normal purpose. The general validity of these results has received an amount of indirect support in the following way.

The photographs of the motion of the ball charge, published by Rose and Evans,(9) have been questioned on the grounds that they show no cataracting. A study of the original article shows, however, that the three cases are $J=0{\cdot}25$, $J=0{\cdot}5$ and $J=0{\cdot}75$, with $N/N_c=0{\cdot}56$ in each case. From Table 2.1, however, it is seen that each of these values of $J$ is above the maximum for which cataracting will be apparent, and so the results shown in the photographs are to be expected.

The introduction of a coefficient of friction of constant value, as in the extended theory, is not completely adequate, however, since a study of Fig. 2.13b suggests that the average angle of slope of the surface of the charge should decrease with increasing filling, whereas the

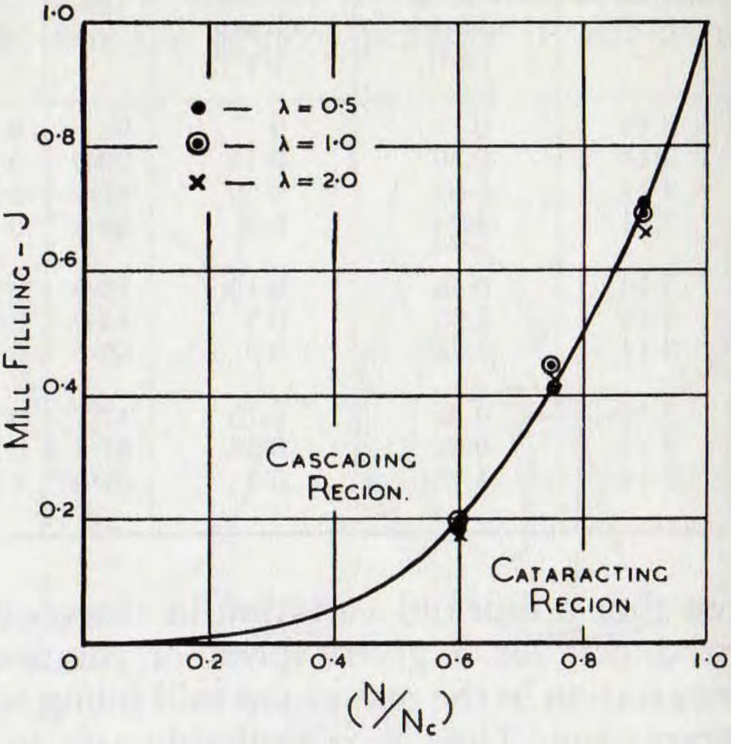

Fig. 2.19

measurements of Rose and Evans(9) show that the slope increases with mill filling. This difference is easily explained since the charge consists of a limited number of balls of finite size and, in consequence, the energy dissipated in friction in the charge increases with the size of the charge. This has an effect equivalent to an increase in the coefficient of friction of the charge; and so brings about an increase in the slope of the surface of the charge, in the same way as is shown in Fig. 2.13a. Thus, it appears that the introduction of a variable value for the coefficient of friction, $\lambda$, would bring about an even better agreement between theory and practice. Unfortunately, the introduction of such a variable quantity very much complicates the mathematical analysis.

A final, and very important, use to which the foregoing constructions can be placed is to ensure that the condition

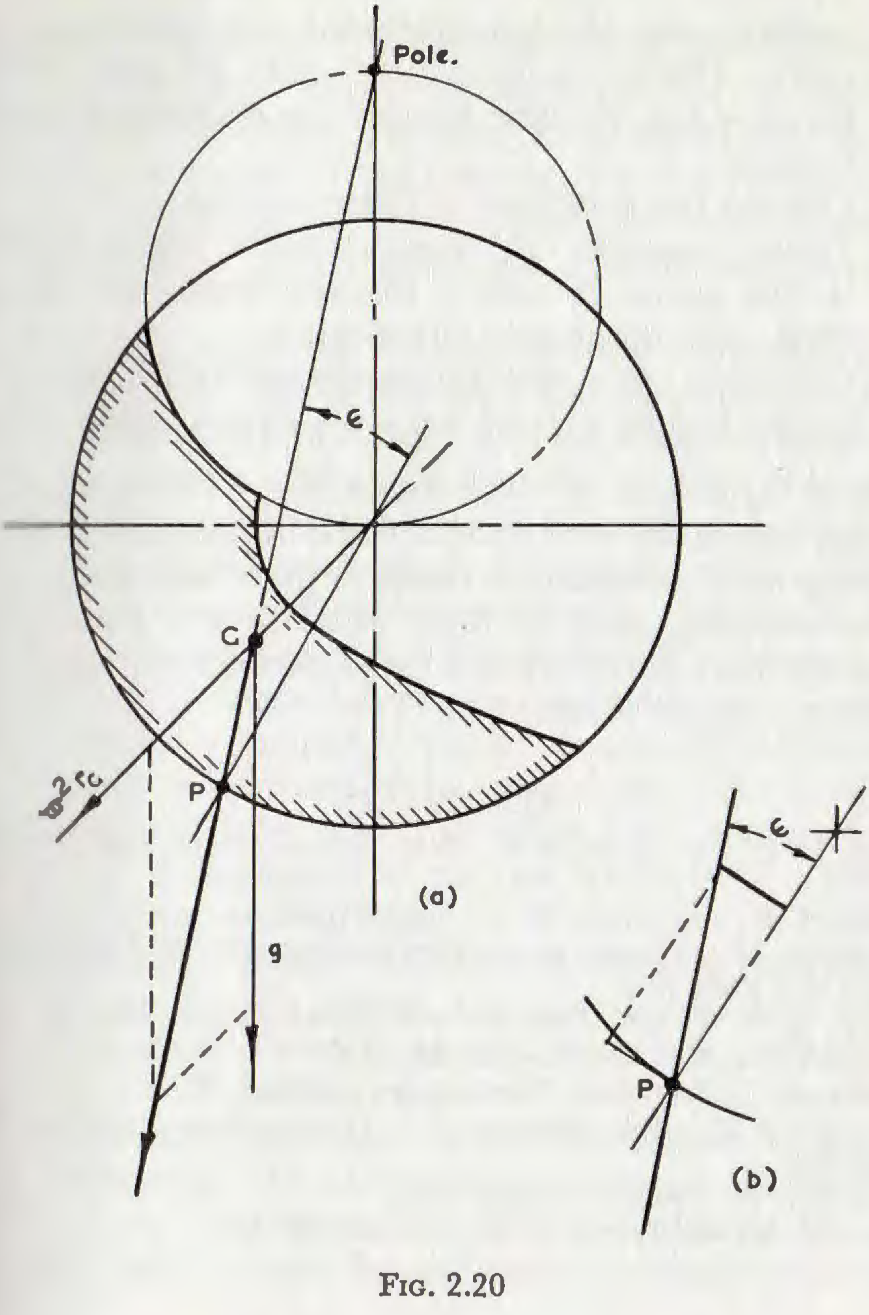

FIG. 2.20

that the charge shall not slip on the mill lining is fulfilled. For this purpose,

(1) Use the previous constructions to determine the configuration of the "rolling" charge in the mill; shown shaded in Fig. 2.20a.

(2) Determine the position of the centre of gravity, G, of this area.

(3) Through G plot the radial acceleration $\omega^2 r_G$ radially and the gravitational acceleration $g$ vertically. (Note: radial force $= M r_G \omega^2$ and vertical force $= Mg$, so these vectors are proportional to the forces.)

(4) Obtain the resultant of these vectors.

(5) Draw a straight line connecting the centre of mill O to the point P, where the resultant cuts the mill shell, and determine the angle $\varepsilon$.

(6) Compare tan $\varepsilon$ with the coefficient of friction $\lambda$. For safety $\lambda$ must exceed tan $\varepsilon$ by a reasonable margin.

The last condition is clear from Fig. 2.20b, since the resultant has radial and tangential components as shown. The tangential component tends to give rise to slip at the surface and slip will, in fact, occur if the ratio of the tangential force to the radial force exceeds the coefficient of friction: that is if $\tan \varepsilon > \lambda$.

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## CHAPTER 3

# THE POWER REQUIRED TO DRIVE A MILL

The power required to drive a tumbling mill is of interest both to the designer and to the mill operator: to the former as a basis of design for the determination of the necessary size of the elements of the machine; and to the latter because, all other factors being equal, the most economical machine is that which demands a minimum power for driving.

The power required to drive a mill depends, to some extent on every one of the physical dimensions defining the mill shell and ball charge and on many of those defining the properties of the powder charge. Thus the number of variables involved is very large. Since even a moderately complete theory for the internal dynamics of the ball mill, in which all these variables are given due importance, has not been propounded, the calculation of the power requirements of a mill, from theoretical considerations, cannot be made. Similarly, owing to the great number of variables, no complete experimental investigation of the power demands of ball, tube and rod mills has been made, the amount of work required for an analysis of all of the variables being prohibitive.

Probably one of the most comprehensive experimental investigations of the power demands of the ball or tube mill is that carried out by one of the present authors, with a co-worker, Rose and Evans,(1) by use of small-scale models; the test apparatus being that shown in Fig. 3.1. By use of small-scale models, a very large number of tests may be carried out in a reasonable time and, furthermore, variables which would be difficult to change on a large-scale test may easily be studied. No valid major objection to the use of small-scale models exists since by use of the method of dimensional analysis the results may be generalized to be applicable to mills of any size. The valid generalization of the results of the model test demands that there should be a complete dynamical similarity between the systems, and, in the case of the ball mill, the pressures in the bed of material in the mill cannot be made strictly similiar in a large and a small mill. Even so the satisfactory results obtained by use of model tests suggest that errors arising from this source are small. In the present chapter the question of the power to drive a mill will be approached from the point of view of these model tests and the results of previous workers studied by comparison with these data.

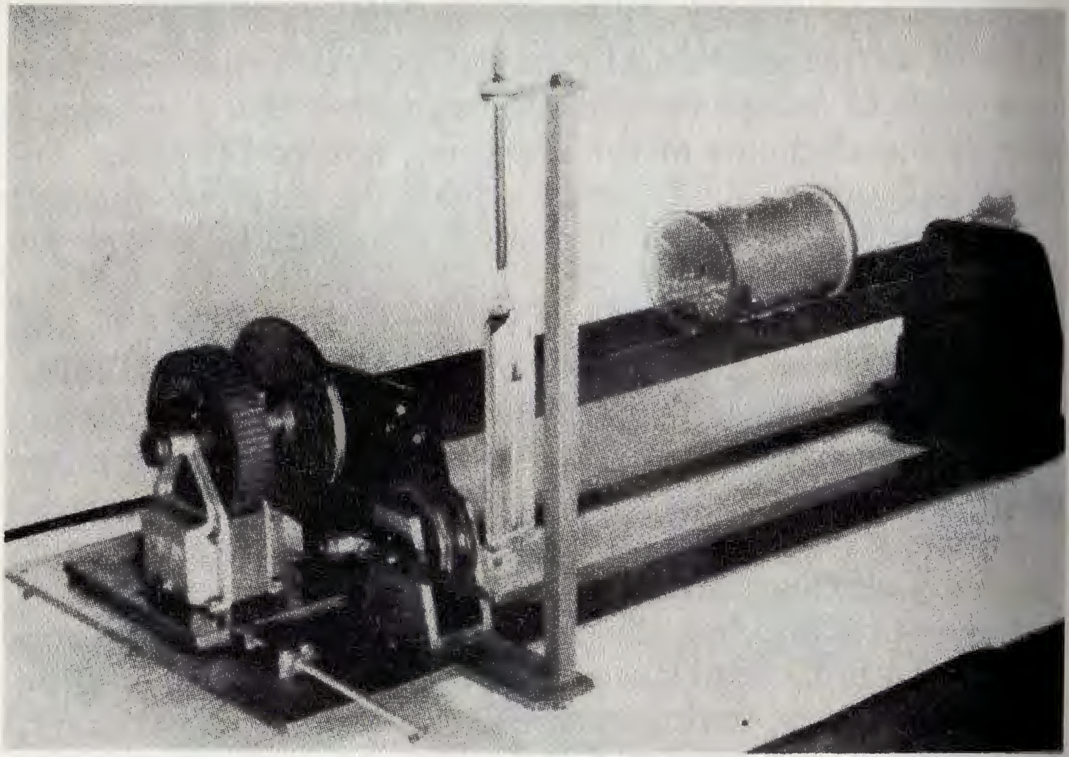


Fig. 3.1
Apparatus for Tests on small-scale mills

The power, $P$, to drive the mill would be expected to depend upon the length of the mill, $L$, the diameter, $D$, the diameter of ball, $d$, the density, $\varrho$, of the ball, the volume occupied by the charge (including voids), expressed as a fraction of the total mill volume, $J$, the speed of rotation, $N$, the acceleration due to gravity, $g$, the coefficient of restitution of the material of the balls and mill, $e$. The power, $P$, would also be expected to depend upon the following characteristics of the powder; the representative diameter of the particles, $b$, the energy necessary to bring about unit increase in the specific surface of the powder, $E$, and the volume, $V$, occupied by the powder charge (including voids), expressed as a fraction of the volume between the balls in the mill. Furthermore, the power, $P$, would be expected to depend upon the effective kinematic viscosity of the mixture of powder and fluid, $\nu$, the effective density of the mixture, $\sigma$, and, in the case of wet milling, by the ratio of the volume of the solid material to the volume of the liquid in the pulp, $U$. Finally, when the interior of the mill is fitted with lifters it would be expected that the power, $P$, would depend on the number of lifters, $n$, and upon the height of the lifters, $h$.

Thus, the power can be expressed symbolically:

$$P = \phi(L, D, d, \varrho, J, N, g, f, e, b, E, V, \nu, \sigma, U, h, n) \quad (3.1)$$

where $\phi$ denotes some function of each of the quantities within the bracket.

Then from the well-known principles of dimensional analysis it is easy to derive the expression in terms of dimensionless groups,

$$\left(\frac{P}{D^5N^3\varrho}\right) = \phi\left\{\left(\frac{h}{D}\right), \left(\frac{L}{D}\right), \left(\frac{d}{D}\right), \left(\frac{g}{DN^2}\right), \left(\frac{b}{D}\right), \left(\frac{E}{D^3N^2\varrho}\right), \left(\frac{\nu}{D^2N}\right), \left(\frac{\sigma}{\varrho}\right), (J), (f), (e), (V), (U), (n)\right\} \quad \ldots\ldots (3.2)$$

where $\phi$ denotes some function of each of the dimensionless groups upon the R.H.S. of the equation.

Theoretically, the dimensionless groups can be combined in any way and it is the function of experiment to determine these relationships. In the present work, however, it has been found that, in most cases, the functions of the dimensionless groups may be multiplied together and the results of the experimental analysis have been arranged in this way.

On this basis equation (3.2) can be written:

$$\left(\frac{P}{D^5N^3\varrho}\right) = \phi_1\left(\frac{g}{DN^2}\right)\cdot\phi_2\left(\frac{L}{D}\right)\cdot\phi_3(J)\cdot\phi_4\left(\frac{d}{D}\right)\cdot\phi_5(n)$$

$$\times\phi_6\left(\frac{h}{D}\right)\cdot\phi_7(f)\cdot\phi_8(e)\cdot\phi_9\left(\frac{b}{D}\right)\cdot\phi_{10}\left(\frac{\sigma}{\varrho}\right)\cdot\phi_{11}(V)$$

$$\times\phi_{12}\left(\frac{E}{D^3N^2\varrho}\right)\cdot\phi_{13}(U)\cdot\phi_{14}\left(\frac{v}{D^2N}\right) \qquad (3.3)$$

In the work of these investigators, attention has been confined to dry grinding and, in this case, the groups $(v/D^2N)$ and $(U)$ are eliminated.

The first variable to be studied was the effect of the length of the mill on the power requirements: that is, the determination of $\phi_2(L/D)$. The experimental results of Rose and Evans fall on a series of straight lines passing through the origin and so it follows that $\phi_2(L/D)$ is a linear function. Thus,

$$(L/D) = K(L/D) \qquad \ldots\ldots (3.4)$$

The critical speed of a mill has already been discussed in Chapter 2 and it has been shown there, by equation (2.5), that the critical speed is given by

$$\omega_c = \sqrt{(2g/D)} \qquad \ldots\ldots (3.5)$$

which, by transposition, gives

$$g/D = KN_c^2 \qquad \ldots\ldots (3.6)$$

If now $N$ is the actual speed of rotation, it follows that

$$g/DN^2 = K(N_c/N)^2 \qquad \ldots\ldots (3.7)$$

or that the group $(g/DN^2)$ is proportional to the square of the ratio of the critical speed to the actual speed of rotation. Thus the relationship may be written:

$$\phi_1(g/DN^2) = \phi_1'(N_c/N) \qquad \ldots . \; (3.8)$$

It also follows that in order that dynamical similarity, as regards this group, shall hold between a number of mills of different diameters, it is necessary that they shall all run at the same fraction of their respective critical speeds. This is in accordance with the findings of Gow, Campbell and Coghill(2) and other workers.

The relationship between the power group $(P/D^6N^2\varrho)$ and the group $(N_c/N)$, as determined by experiment, is shown in Fig. 3.2, and it is seen that for speeds less than 80% of the critical, the relationship $\phi_1'(N_c/N)$ can be replaced by

$$\phi_1'(N_c/N') = K_2(N_c/N)^2 \qquad \ldots . \; (3.9)$$

where $K_2 = 3{\cdot}13$ for lifter mills and 3·66 for mills without lifters. Within this range these relationships can be used directly in the equations for the determination of power required to drive a mill. The deviation from linearity shown by these curves is exactly what would be predicted from theoretical consideration since, ideally, when the whole ball charge is centrifuging, the centre of gravity of the charge would coincide with the axis of rotation and the power required to drive the mill would be zero.

In theory, the centrifuging of the charge takes place in layers and each layer forms at a definite speed. In practice, however, the process is complicated by interaction between the balls, lifters, etc., and centrifuging occurs more or less gradually over a wide range of speeds, this leading to the deviation from linearity, shown in the figure. Since these curves are plotted on logarithmic scales, the curves should be asymptotic to the negative branch of the $y$-axis, and in fact the lines show this trend.

Since, as has already been mentioned, the critical speed is a criterion for the occurrence of many phenomena in a mill, it is perhaps of interest to mention that during the course of tests at various speeds, the configuration of the balls within the shell was observed through a perspex end

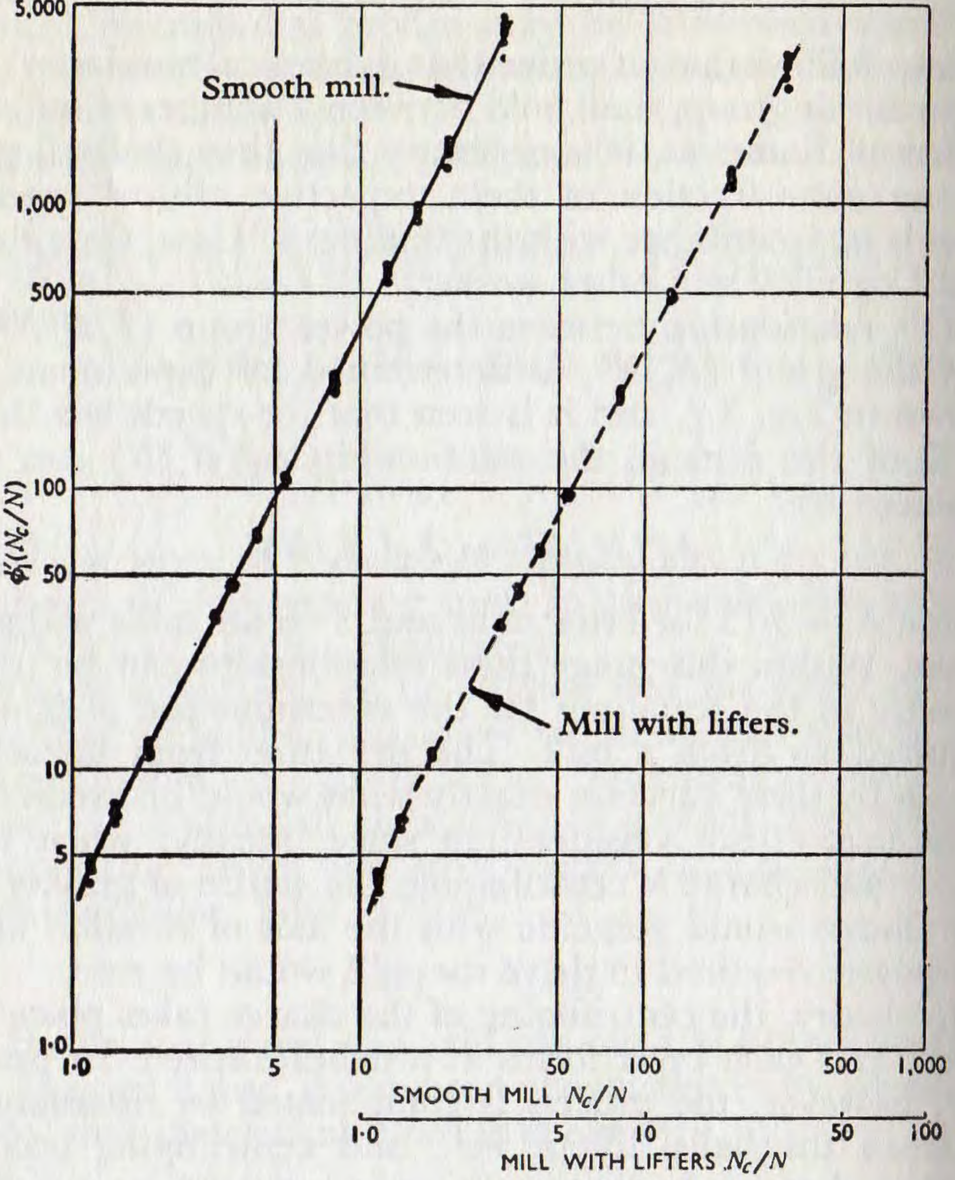


FIG. 3.2

cap, and, in the case of the smooth mill, centrifuging of the outer layer of the balls was not observed at speeds up to $1{\cdot}2N_c$, the highest speed reached during the tests, but when effective lifters were fitted to the shell, the outer layers of balls commenced to centrifuge when the speed of rotation exceeded the critical value $N_c$, by a very small amount. For a smooth mill, cataracting of the charge generally commenced at speeds of about $0{\cdot}8N_c$, while, when lifters were fitted to the shell, cataracting commenced at about $0{\cdot}6N_c$. This is roughly in accord with the results of Chapter 2, illustrated in Fig. 2.19, that if a mill with a 30 % to 40 % filling is taken as representative, the

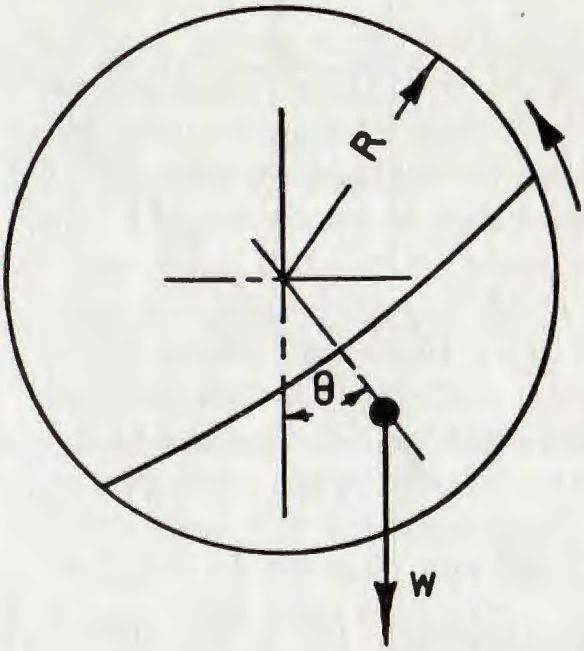
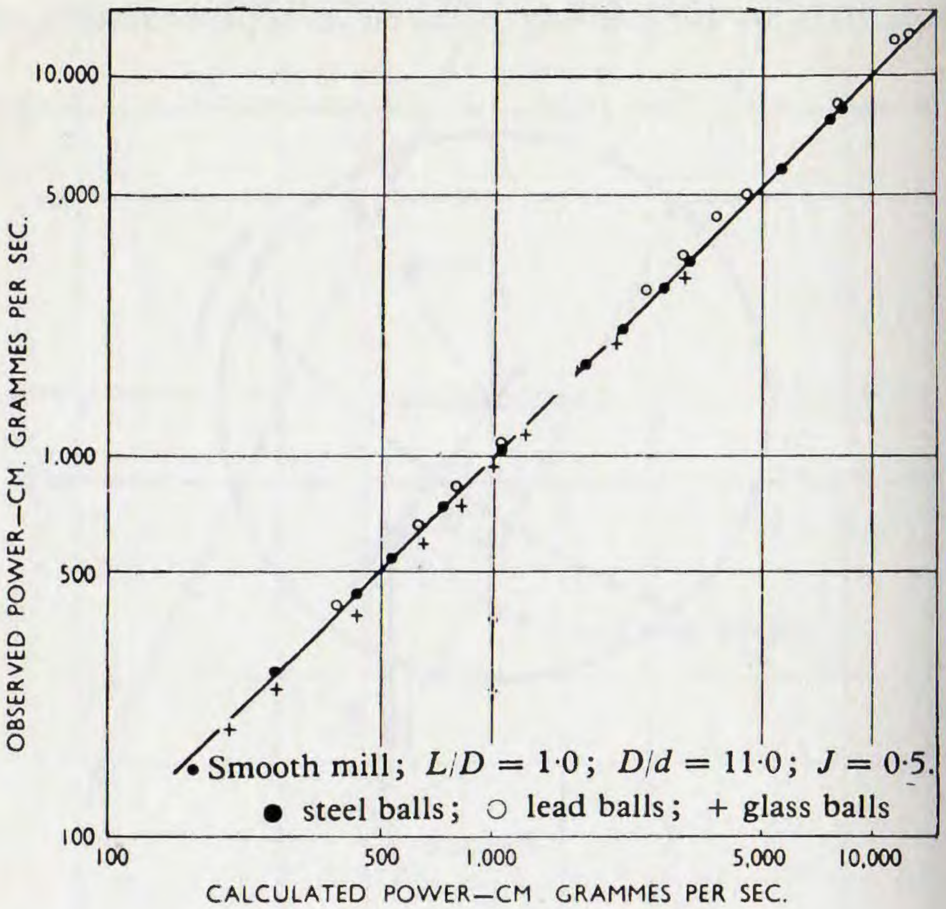


Fig. 3.3

boundary between cataracting and cascading is when the speed of rotation is about $0{\cdot}7N_c$.

With a smooth mill surging of the entire charge was sometimes observed for speeds below about $0{\cdot}4N_c$.

Coming now to a study of the effect of the density of the grinding media upon the power required to drive a mill it follows that since the mass of grinding bodies has a definite configuration, as suggested by Fig. 3.3, then the torque to drive the mill will be proportional to the weight of the charge; that is, without powder, proportional to the density of the material of the balls or rods. The correctness of this view is established by Fig. 3.4a, based on the work of Rose and Evans,(3) in which the measured power is plotted against power calculated on the basis of this assumption for charges of glass, steel and lead balls; the relative densities of these materials being given in Table 3.1. It is at once seen that an extremely good linear relationship holds between these variables and so it must be concluded that the suggested relationship is valid.



(a)

Fig. 3.4

If now the material undergoing grinding occupies the spaces between the balls of the ball charge, it follows that the centre of gravity of the powder mass will coincide with that of the ball mass. Thus, if the mill charge retains the

TABLE 3.1

| Material | Specific gravity |
|---|---|
| Glass | 2·65 |
| Steel | 7·8 |
| Lead | 11·4 |

same configuration as it has when balls only are present, that is, the configuration is unaltered by the inclusion of the powdered material, then it follows that the torque to drive the mill will be proportional to the total weight of the charge in the mill. Thus it would be expected that, at least as a first approximation, the power to drive a mill containing a powder can be calculated from the equation applicable to a mill containing balls only, provided that the density of the balls, in the latter case, is increased by an amount sufficient to include the effects of the additional mass due to the powder charge. If this point of view is adopted, it follows that the mill charged with powder becomes a special case of the mill containing only a ball charge, and effects arising from the particle size and other characteristics of the powder charge may be expressed by corrections to the equations applicable to the mill without powder. On this basis, for a mill containing a powder charge

Power input

= (Torque due to ball space charge + Torque due to powder) × Speed × Proportionality factor

= $K$ × (Wt. of ball charge + Wt. of powder charge)

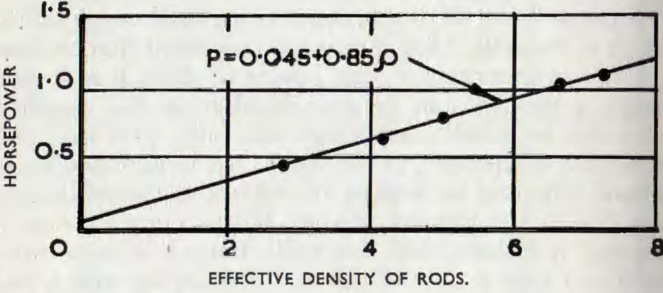
That these terms are simply additive is supported by the curves of Fig. 3.4b in which are plotted data given by Coghill and Devaney,(4) for the power required to drive a rod mill, in which the rods are tubular and the effective density of the rods is varied by "filling" the tubes with various materials. For these tests the mill was grinding chert, for which $\sigma \simeq 3{\cdot}0$, and, since the void space between the rods is about 15 % of the mill filling, it follows that, if the simple addition is valid, the power is given by the expression of the form

$$y = mx + c$$

Furthermore, the power is given by

$$P = K(0{\cdot}015 \times 3{\cdot}0 + 0{\cdot}85\varrho)$$

and it is seen that the curve given coincides very closely to this expression.



(b)

FIG. 3.4

If now $\xi$ is the ratio of the power to drive the mill charged with balls and powder to the power to drive the same mill with a ball charge only, then:

$$\xi = \frac{\text{Wt. of ball charge} \times \text{Wt. of powder charge}}{\text{Wt of ball charge}}$$

+ ×?

If also it is assumed that the porosity of the ball charge and of the powder is 0·4 in each case, then:

$$\xi = \frac{(1-0{\cdot}4)\varrho + 0{\cdot}4(1-0{\cdot}4)\sigma}{(1-0{\cdot}4)\varrho}$$

$$= (1 + 0{\cdot}4\sigma/\varrho) \qquad \ldots\ldots \quad (3.10)$$

$1 + 0{,}4 \times 2{,}7/7{,}8 \cong 1{,}14$

$1 + 0{,}4 \times 2{,}6/7{,}8 \cong 1{,}13$

Thus the functional relationship $\phi_{10}(\sigma/\varrho)$ is given by the multiplicative factor $(1+0{\cdot}4\sigma/\varrho)$.

The validity of this relationship has been further examined by tests carried out using powders consisting of emery, iron powder and silica sand; the density of these materials being given in Table 3.2.

TABLE 3.2

| Material | Density (g/cm$^3$) |
|---|---|
| Iron | 7·9 |
| Emery | · 95 |
| Silica sand | 2·7 |

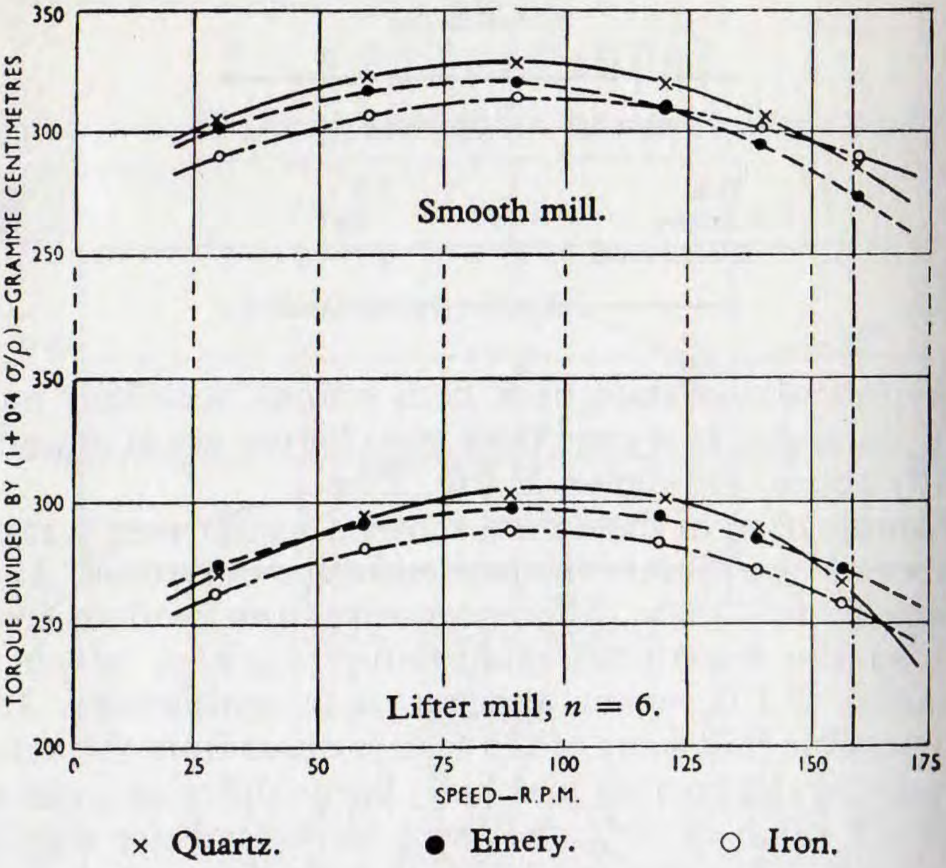
3.95 (3.9-4.0)

Representative data from both smooth and lifter mills, after reduction to a consistent basis by the use of equation (3.10) above, are shown in Fig. 3.4c.

Examination of these data shows a scatter over a range of about 5 % with no consistencies in the deviations. Thus, it appears that these differences arise from random errors and so the functional relationship $\phi_{10}(\sigma/\varrho)$, given by equation (3.10), again appears to be satisfactory. It is also possible that some of the scatter arises from the actual porosity of the powder and ball charge differing from the assumed value of 40 %, but such deviations are scarcely calculable. Thus it is concluded that the function $\phi_{10}(\sigma/\varrho)$ may be replaced by $(1+0{\cdot}4\sigma/\varrho)$, with an accuracy sufficient for practical purposes. The remaining functional relationships cannot be represented by simple algebraic expressions and so are best presented by graphs.

In the first place it follows from Fig. 3.4a that, since materials having a wide range of values of coefficient of restitution are used and yet the points suffer no significant scatter, the effect of the coefficient of restitution on the power requirements of the mill is very small. Thus, at least for practical purposes, the function $\phi_8(e)$ may be neglected.

This result would be expected since, although the coefficient of restitution of the metals would affect the conditions of rebound when clean solid particles make contact, the effects of changes in the coefficient of restitution of the metals are, in the case of a ball in a mill, masked by the energy dissipating capacities of the loose bed of balls and powder upon which the ball falls and by powder between the contact surfaces.



(c)

Fig. 3.4

If, on referring to Fig. 3.3, it is assumed that the curve representing the free surface of the charge is a straight line and also that the angle $\theta$ is unaltered by variation of the quantity of the charge, then it is clear that the power to drive the mill is proportional to the product of the cross-sectional area of the charge and the radius of the couple arm; both of which depend on the mill filling $J$. On the basis of these assumptions it is a simple matter to calculate the torque in terms of the mill filling and, when this is done, the "theoretical" line of Fig. 3.5 is obtained. This method of treatment is a gross over-simplification, however, since, as explained in Chapter 2, the curve representing the surface of the charge depends on the filling and frictional characteristics of the material. Furthermore, the balls are not mathematical points and so there is mutual

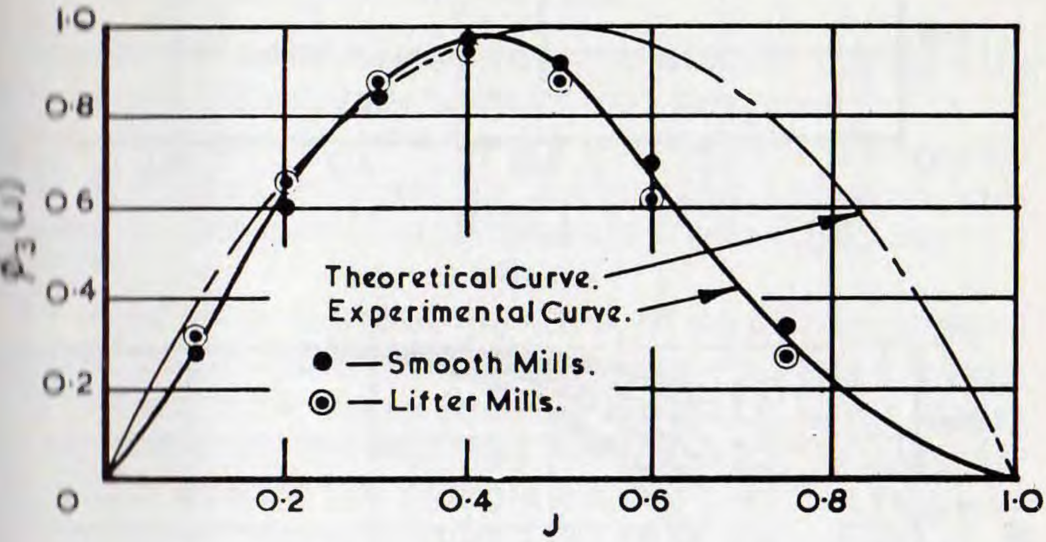


FIG. 3.5

interference between the balls. There is also interference between the balls and lifters, etc., and, when the relationship is determined by experiment, a vastly different curve, as is also shown in Fig. 3.5, is obtained. The importance of this difference will become apparent later, when various simplified formulae, which have been proposed for the power requirements of a mill, are discussed.

It has already been explained in Chapter 2, in which the motion of a mill charge is discussed, that the coefficient of friction between the ball surfaces and the number of balls in the charge are, from the point of view of motion of charge, not independent variables. For this reason the functions $\phi_4(d/D)$ and $\phi_7(f)$ must be considered together. In Fig. 3.6a are shown the results of tests on a smooth mill containing clean steel balls and in Fig. 3.6b of tests

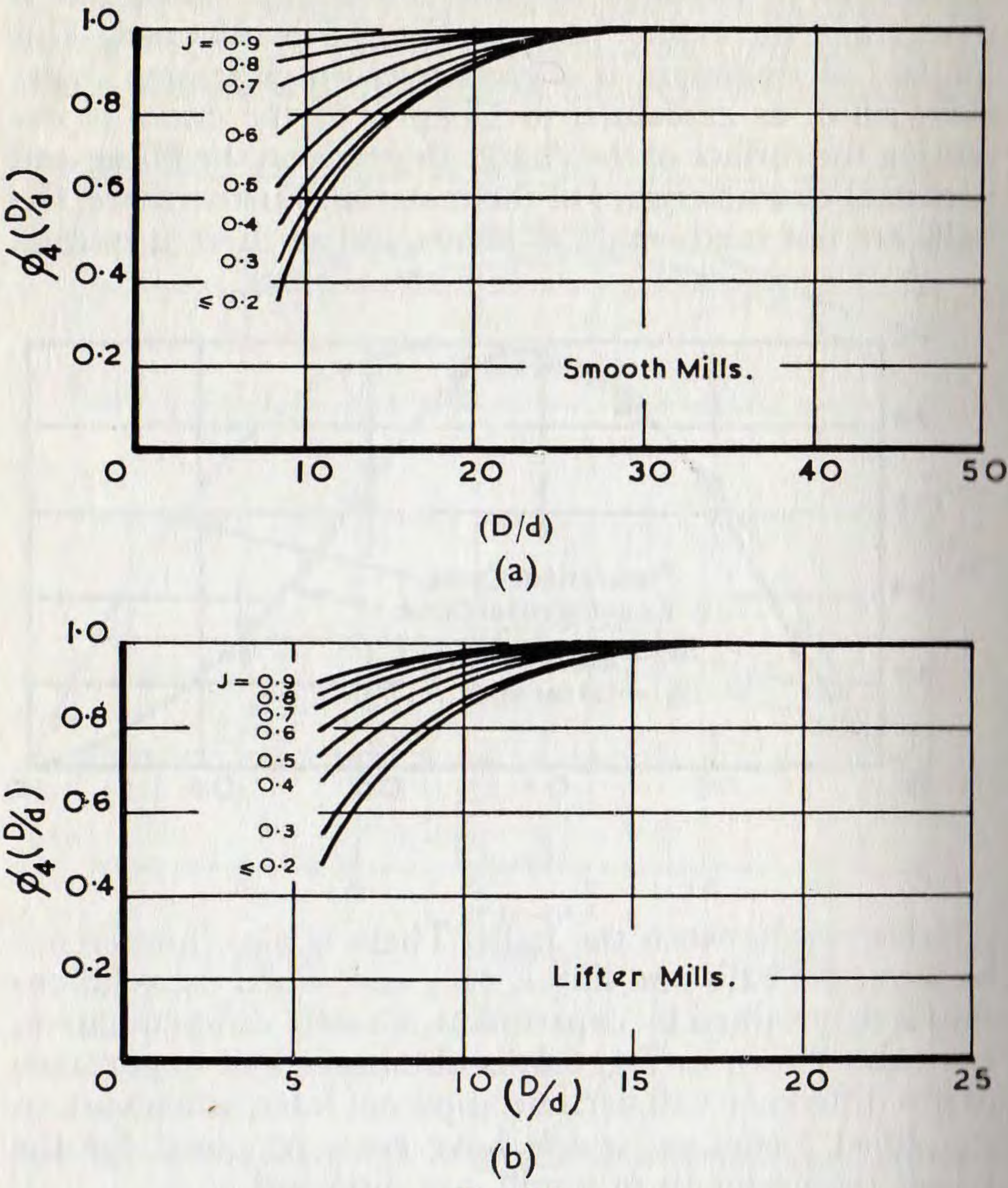


FIG. 3.6

on a mill with lifters. Examination of these curves shows that, for the practical case of $(D/d)$ greater than 20, the value of the function does not differ significantly from unity. For values of $(D/d)$ appreciably less than 20, however, the value of the function is not independent of the filling of the mill.

In order to test the effects of variation of the coefficient of friction tests have been carried out in which:

(1) The balls and mill are cleaned and tested dry.
(2) The balls and mill are cleaned and slightly oiled.
(3) The balls and mill are cleaned and treated with molybdenum disulphide.
(4) The balls and mill are contaminated with a trace of powder.
(5) A full powder charge is used.

The coefficient of friction between surfaces treated with molybdenum disulphide is about 0·05 as against 0·1 to 0·2 for oiled surfaces and 0·3 for dry steel on steel. The coefficient of friction between the surface contaminated with powder is unknown but is probably very high, say 0·5 to 1·0.

In order to investigate the effect of small quantities of powder, quartz powder from the sample passing through a 200-mesh sieve was added to the ball charge under investigation and normal tests carried out. The quantity of powder used for this purpose was 5·0 % of that required to fill the void spaces between the balls. This quantity of charge was sufficiently small to ensure that, for all practical purposes, the change in torque, from the corresponding case of the mill charged with clean dry balls only, was due to the change in the coefficient of friction and not to the increased density of the charge. As a result of these tests, no significant difference was found between the results for the cases (1), (2) and (3) above, and the combined results for these tests can be presented by the full line of Fig. 3.7, this curve, of course, corresponding to that for $(J) = 0{\cdot}5$ on Fig. 3.6. It is probable that this drooping characteristic arises from the comparative freedom of rotation of the balls when the coefficient of friction is low and when the number of contact points between the surfaces is small; as is bound to be the case when $(D/d)$ is small.

The effect of adding a small quantity of powder is shown by the broken line on Fig. 3.7, this curve being adequate for all values of $(J)$ and, furthermore, a study of Fig. 3.6 suggests that this curve is adequate for lifter mills. Thus, it may be concluded that for smooth mills grinding materials having a low coefficient of friction $\phi_4(D/d)$ should be read from Fig. 3.6a. For lifter mills, or mills grinding materials giving a high coefficient of friction, the functional relationships should be read from the dotted line of Fig. 3.7.

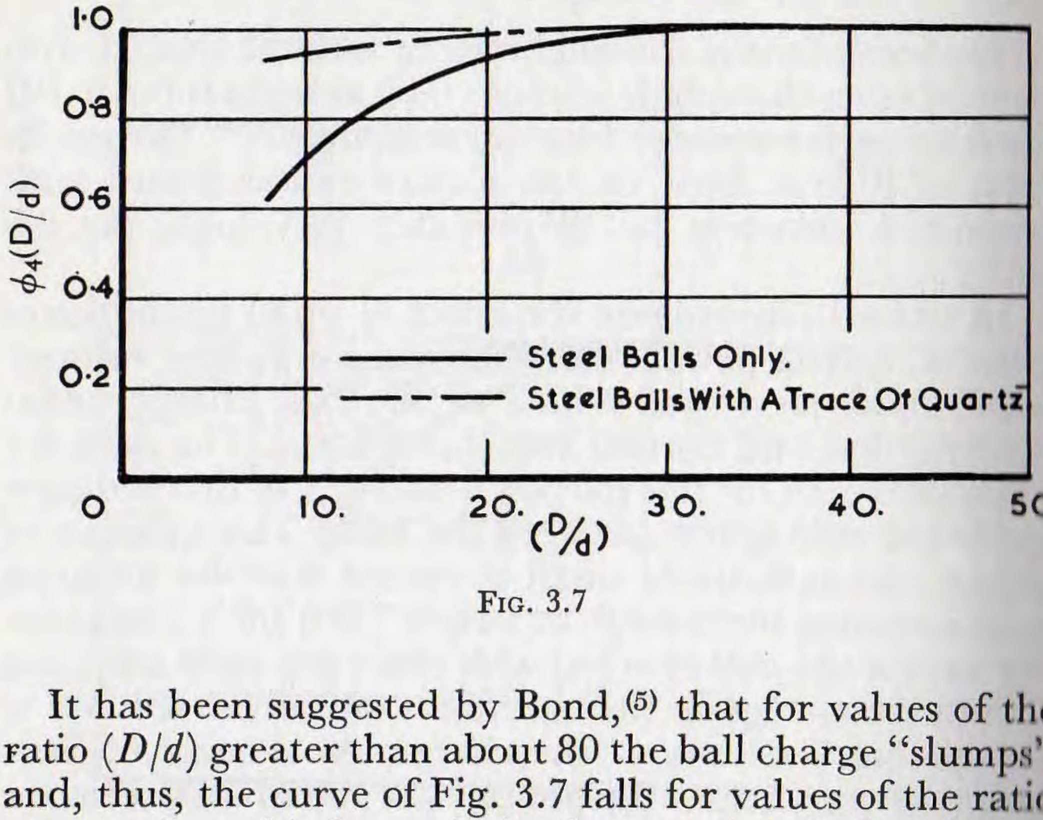


FIG. 3.7

It has been suggested by Bond,⁽⁵⁾ that for values of the ratio $(D/d)$ greater than about 80 the ball charge "slumps" and, thus, the curve of Fig. 3.7 falls for values of the ratio greater than this figure. It is also stated that this effect is particularly marked in wet milling, but, since it is rather unusual to use balls so small that the ratio of $(D/d)$ exceeds 80, it is probable that this effect is not of great practical importance.

In view of this comment the present authors have, by use of the small-scale apparatus, subsequently extended the range of Fig. 3.7 to $(D/d) \simeq 200$ and, in no case, with mills charged with iron shot or glass balls, has "slumping" of the charge been observed during dry milling.

Thus, no explanation of this phenomenon can be offered but it is probable that it is in some way bound up with a variation in the frictional characteristics of the mill charge.

The results of tests carried out to determine the effect of the number of lifters, *n*, on the power requirements to drive a mill, that is to establish $\phi_5(n)$, are plotted in Fig. 3.8. The function $\phi_5(n)$ is influenced by a large number of variables to a small extent, and the curves presented in this figure are the lines best representing the whole of these data. This method of presentation is adopted since it is felt that the errors introduced by the use of a single line, to replace many, are not significant but that the simplification obtained is worthwhile.

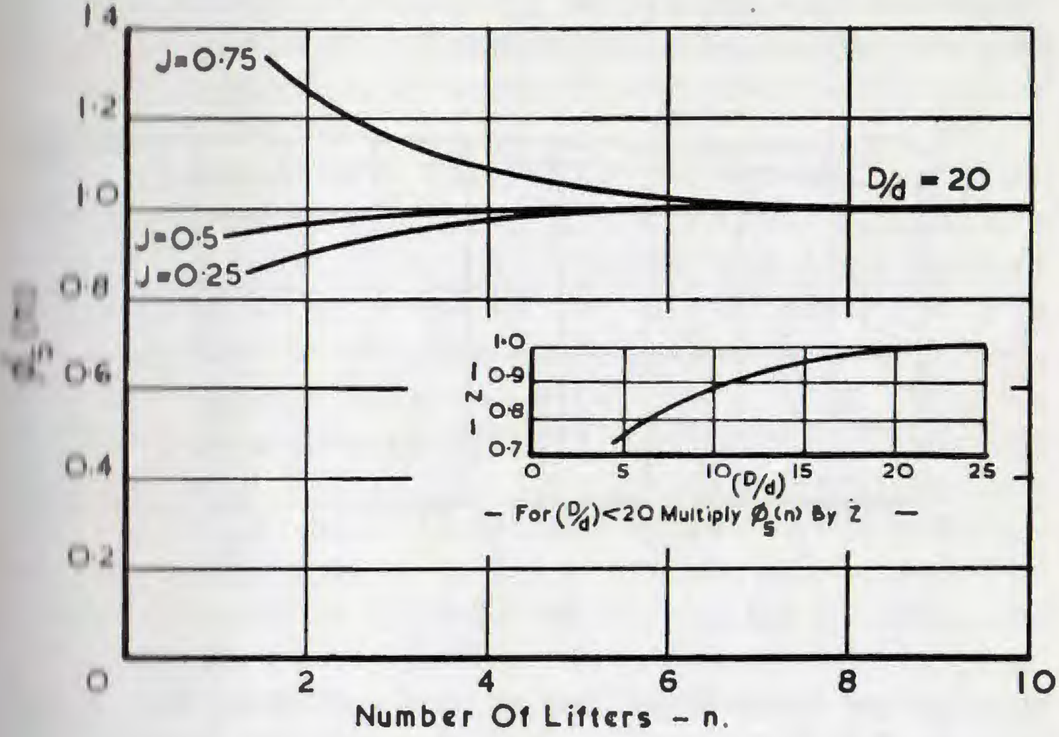


FIG. 3.8

From the figure it can be seen that, provided the number of lifters is greater than about 6 and the ratio of (*D*/*d*) exceeds about 20, the function has a constant value of unity. Thus it follows that, provided there are sufficient lifters to effectively key the charge to the mill shell, the number of lifters is unimportant. For a number of lifters less than 6, but with $(D/d)$ less than 20, there is some dependence upon the value of mill filling as shown by the branching of the curve to the left of the figure. For practical purposes the value of the function, for values of $(D/d)$ less than 20, can be calculated by multiplying the relevant value read from the main curve of Fig. 3.8 by the appropriate correction factor read off from the graph inset in that figure.

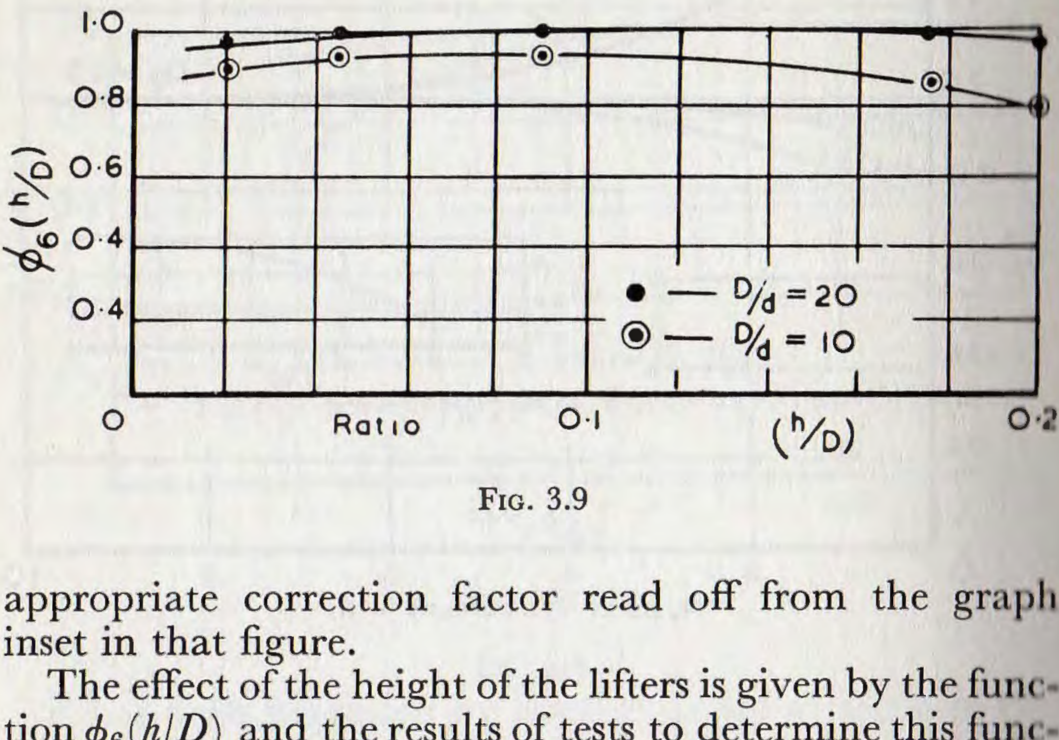


FIG. 3.9

The effect of the height of the lifters is given by the function $\phi_6(h/D)$ and the results of tests to determine this function are given in Fig. 3.9, from which it is seen that for mills in which $(D/d)$ is greater than about 20, the height of the lifter has little effect on power requirements. For the less usual case, in which $(D/d)$ is less than 20, there is a considerable deviation for these limited cases, but a straight-line interpolation between the two curves of Fig. 3.9 is probably adequate. These conclusions are valid only for lifters of reasonable height, that is, for lifters in which $h$ is not greater than, say, $0{\cdot}2D$ and not less than about $d/2$. If $h$ exceeds $0{\cdot}2D$ the lifters unduly interfere with the motion of the ball charge, while if $h$ is much less than $d/2$ the balls ride over the lifters and so locking to the shell does not occur. This at once suggests that "wave", "ship-lap", and other such liners cannot act as effective lifters.

The effect of the particle size on the power requirements of a mill would be expected to be small, provided that the particles were not so small that powerful aggregation takes place. If this were to occur, the charge could become "jelly-like" and the motion of the ball charge would be seriously impeded; with a consequent effect on the power demands of the mill.

The results of tests, carried out on smooth and lifter mills, to determine the form of the function $\phi_9(D/b)$ are given in Fig. 3.10; these tests being such that the value of the group $(D/d)$ exceeded 19, $(J)$ varied from 0·1 to 0·75 and in the case of the lifter mill, $(h/D)=0{\cdot}1$ and $(n)=6$, whilst the group $(D/b)$ was varied from 77 to 740; this range of values of the group $(D/d)$ largely representing the practical range covered by the operating conditions of a large number of mills. The points relating to different test conditions are seen to suffer a considerable amount of scatter. This scatter probably arises from the interdependence of the variables which has been discussed earlier, and it is felt that, unless a very great complication in the form of additional correction factors is introduced, this must be accepted. For practical purposes such complication is not justified, and, in the present work, mean curves have been drawn through the points of Fig. 3.10. When this is done, the curves for the two cases are seen to be identical and give the functional relationship $\phi_9(D/b)$ shown in Fig. 3.11.

Decreasing the value of the $(D/b)$ ratio in a given mill is seen to decrease the power necessary to drive the mill. This may be explained on the basis that the larger particles separate adjacent balls to a greater extent than do the smaller particles, and, therefore, increase the dilatation of the charge. The centre of gravity of the combined charge is thus displaced towards the vertical through the axis of

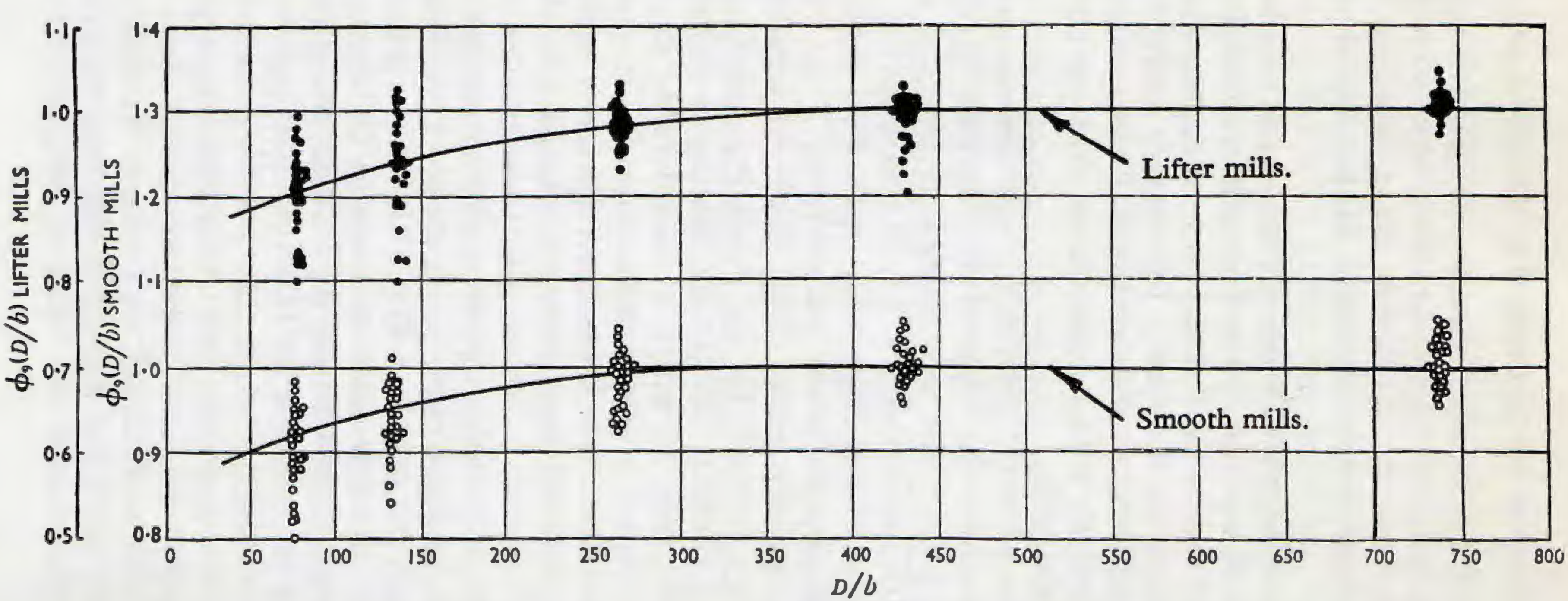

1·1
1·0
0·9
0·8
0·7
0·6
0·5
$\phi_9(D/b)$ LIFTER MILLS
1·4
1·3
1·2
1·1
1·0
0·9
0·8
$\phi_9(D/b)$ SMOOTH MILLS
Lifter mills.
Smooth mills.
0
50
100
150
200
250
300
350
400
450
500
550
600
650
700
750
800
$D/b$

FIG. 3.10

rotation and so the torque is decreased. In the small mills used for these tests, the presence of very fine powder, such that the ratio $(D/b)$ greatly exceeded 800, resulted in separation of the powder from the balls, but this only occurred at speeds in excess of $0{\cdot}8N_c$. The powder formed a continuous layer around the cylindrical wall of the shell, and the power required to drive the mill decreased by a large amount when this occurred. This phenomena would probably not occur with large industrial mills, but data is not available on this point. The question is bound up with the absolute size of the particle and a ratio of $(D/b)$ of 800 in mills used in the tests corresponds to a charge consisting of particles of 0·003-in. average diameter. Generally, however, it would be expected that, except in the case of very fine grinding, this effect is unimportant.

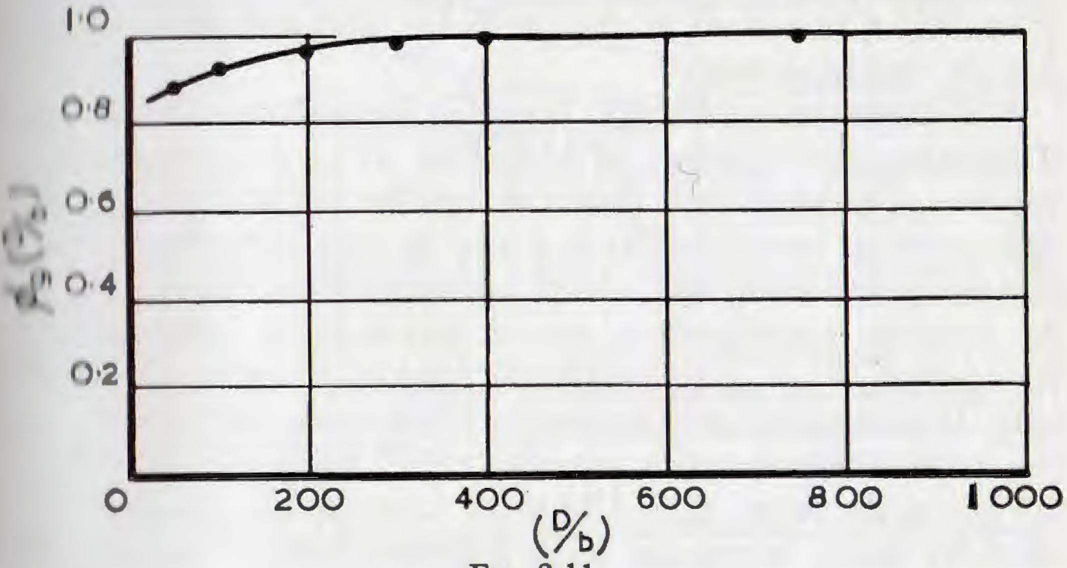


Fig. 3.11

Throughout the series of tests, the value of the powder filling group $(V)$ has mainly been held constant at unity; this being the condition that the powder charge is theoretically just sufficient to fill the void space between the balls. In fact, the powder increases the total volume of the charge and a layer of balls may occur at the surface of the charge, the void space between which is unfilled with powder. This restriction on the value of the parameter $(V)$

has been made because, in practice, less powder than is necessary to fill the void space is rarely used since a part of the ball charge is then inoperative in the grinding process; even though the power necessary to maintain the centre of gravity of the inoperative portion of the ball charge in the displaced position must still be supplied. Conversely, if the powder is excessive, choking of the mill occurs, and a portion of the powder charge is remote from the ball charge at any instant and so cannot be effectively ground. Thus the decision to restrict the value of the parameter $(V)$ to unity is based upon the practical importance of this case and the necessity, in view of the large number of variables involved in the problem, to limit the amount of experimental work.

Although this restriction is useful from the point of view of limiting the number of variables to be investigated, it imposes the restriction that the results are strictly applicable only to batch mills and not to the industrially important grate mills and overflow mills. Thus the results of the present investigation, which has been carried out for the case of $(V) = 1{\cdot}0$, are strictly applicable only to a batch mill. The results have, however, been extended directly to the cases of the trunnion overflow mill and by inference to the grate discharge mill. Consider now the extension of the work to these cases; the mills together with a batch mill have already been illustrated in Fig. 1.4.

With a batch mill, as previously suggested, the centre of gravity of the ball charge and powder charge sensibly coincide and so the density correction $(1+0{\cdot}4\sigma/\varrho)$ previously deduced is applicable.

In the grate mill the powder surface probably slopes from the inlet end of the mill to the discharge end in the manner indicated in Figs. 1.4b and 1.4d. In this case the moment of the powder charge about the axis of rotation of the mill is probably much the same as in the ball mill; the greater moment at one end of the mill being roughly balanced by a smaller moment at the other end. Thus, the method of the present work would be expected to be applicable to a grate mill with an accuracy sufficient for most purposes.

In the case of the overflow mill, in order that the product shall overflow through the hollow trunnion, it is necessary that the general level of the powder surface must correspond to about a 45–50% filling of the mill Then when the ball filling is about 45–50% the overflow mill corresponds to a batch mill, and so the power input should be directly calculable from the present work. For small values of ball charge, however, a correction factor is necessary and this has been determined by experiment and is given in Fig. 3.12. In the use of this factor the power is calculated on the basis that the mill has a ball charge equal to that in the actual overflow mill and the normal correction for powder charge is introduced. This calculated power is then multiplied by a factor, corresponding to the value of the actual ball charge; the factor being read from Fig. 3.12. The shape of the curve of Fig. 3.12 would be expected, since for very small ball charges the powder

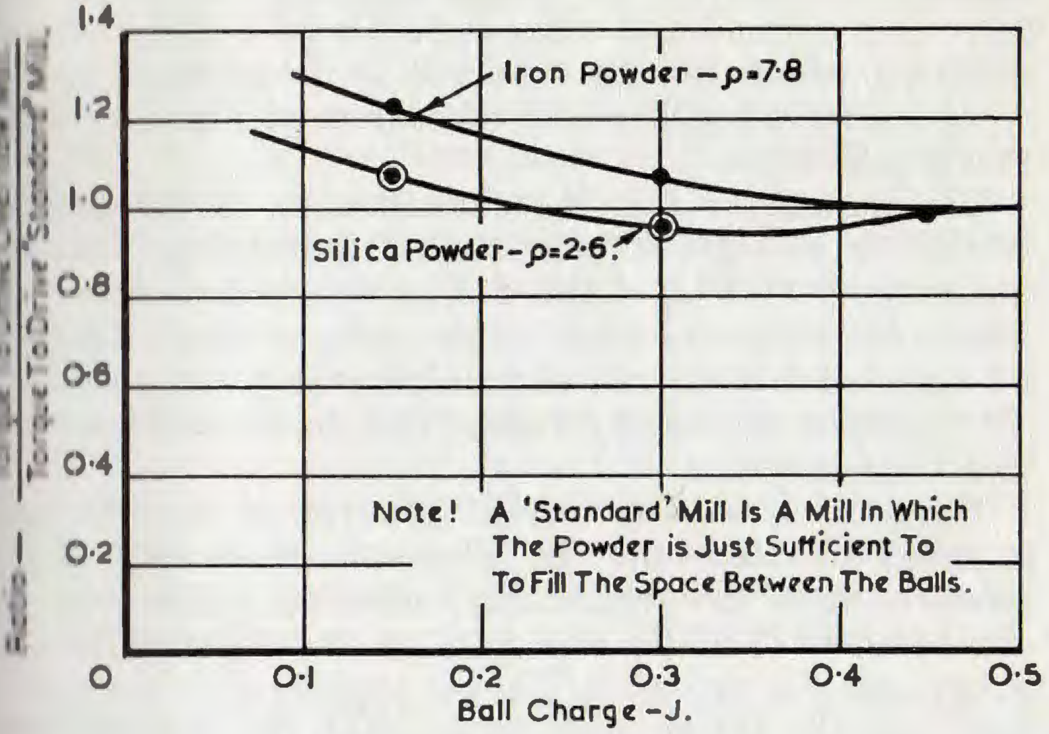


FIG. 3.12

standing above the ball charge level contributes a relatively large moment, and so increases the power required over that calculated on the basis of a batch mill. For intermediate fillings, however, the contribution to the moment by this additional powder is small, but there is, in addition, a loss of moment due to this additional powder displacing the centre of gravity of the relatively heavy ball charge towards the centre of rotation.

Strictly, there should be a correction curve for each value of the density of the charge material, but, in fact, the correction is small and when regard is paid to the overall accuracy which can be expected in calculations of this type, interpolation between the curves of Fig. 3.12 is all that is justified.

The parameter $E$ is the characteristic of the powder which is a measure of its resistance to grinding. Thus the quantity $(E/D^3N^2\varrho)$ would not be expected to affect the instantaneous power input to the mill, although it would be expected that the rate of grinding is dependent upon it. The question of rate of grinding will, however, be treated in a later chapter.

The tests carried out with different materials in the mill, in order to determine the effect of the density of the powder, show no significant variations which can be attributed to the difference in the value of the parameter $E$ between the materials. Thus it appears that in connection with the power input to the mill, the dimensionless group $(E/D^3N^2\varrho)$ is not a significant variable, and so $\phi_{12}(E/D^3N^2\varrho)$ is eliminated from the equation.

In order to compute the power required to drive a mill it is necessary to include the values of all the various relevant factors previously discussed and on the basis of this foregoing work, the equation for the power becomes

$$\left(\frac{P}{D^5N^3\varrho}\right) = \left(1+\frac{0{\cdot}4\sigma}{\varrho}\right)\cdot\left(\frac{L}{D}\right)\cdot\phi_1'\left(\frac{N_c}{N}\right)\cdot\phi_3(J)\cdot\phi_4\left(\frac{d}{D}\right)\cdot\phi_5(n)$$
$$\times\,\phi_6\left(\frac{h}{D}\right)\cdot\phi_9\left(\frac{b}{D}\right)\cdot\phi_{11}(V) \qquad \ldots . \quad (3.11)$$

where $\phi_1'(N_c/N)$ is read from Fig. 3.2,
$\phi_3(J)$ is read from Fig. 3.5,
$\phi_4(d/D)$ is read from Fig. 3.6 or Fig. 3.7 as required by mill parameters,
$\phi_5(n)$ is read from Fig. 3.8,
$\phi_6(h/D)$ is read from Fig. 3.9,
$\phi_9(b/D)$ is read from Fig. 3.11,
and $\phi_{11}(V)$ is read from Fig. 3.12 for a trunnion overflow mill or is taken as unity for a batch mill or grate discharge mill.

The equation (3.11) has been applied to the computation of the power requirements of large industrial mills, based on the published data given by Taggart,(6) and the results of these calculations are shown in Fig. 3.13. From this curve it is seen that the points are scattered along good straight lines, even though the ratio of the horse-power to drive the largest mill to that required to drive the model is of the order of 30,000 to 1·0. Thus it appears that there are no systematic errors in the results of the model tests.

In connection with the scatter of the points, it is perhaps of interest to mention that the data quoted by Taggart, which have been apparently obtained by circulating a questionnaire to a large number of mill owners, are, for the following reasons, subject to some uncertainties.

(1) The power quoted is the power input at the motor terminals. Furthermore, this figure is often estimated; or even the nominal installed power is quoted.
(2) The overall efficiency from the motor terminals to the mill body is unknown and can vary from mill to mill. For purposes of calculations an overall efficiency of 85 % has been assumed for mills with concentric drives, and an overall efficiency of 75 % for mills with girth ring drives.
(3) The figures for diameter and length are quoted on various bases; for example some are the nominal dimensions of the mill and others are inside new

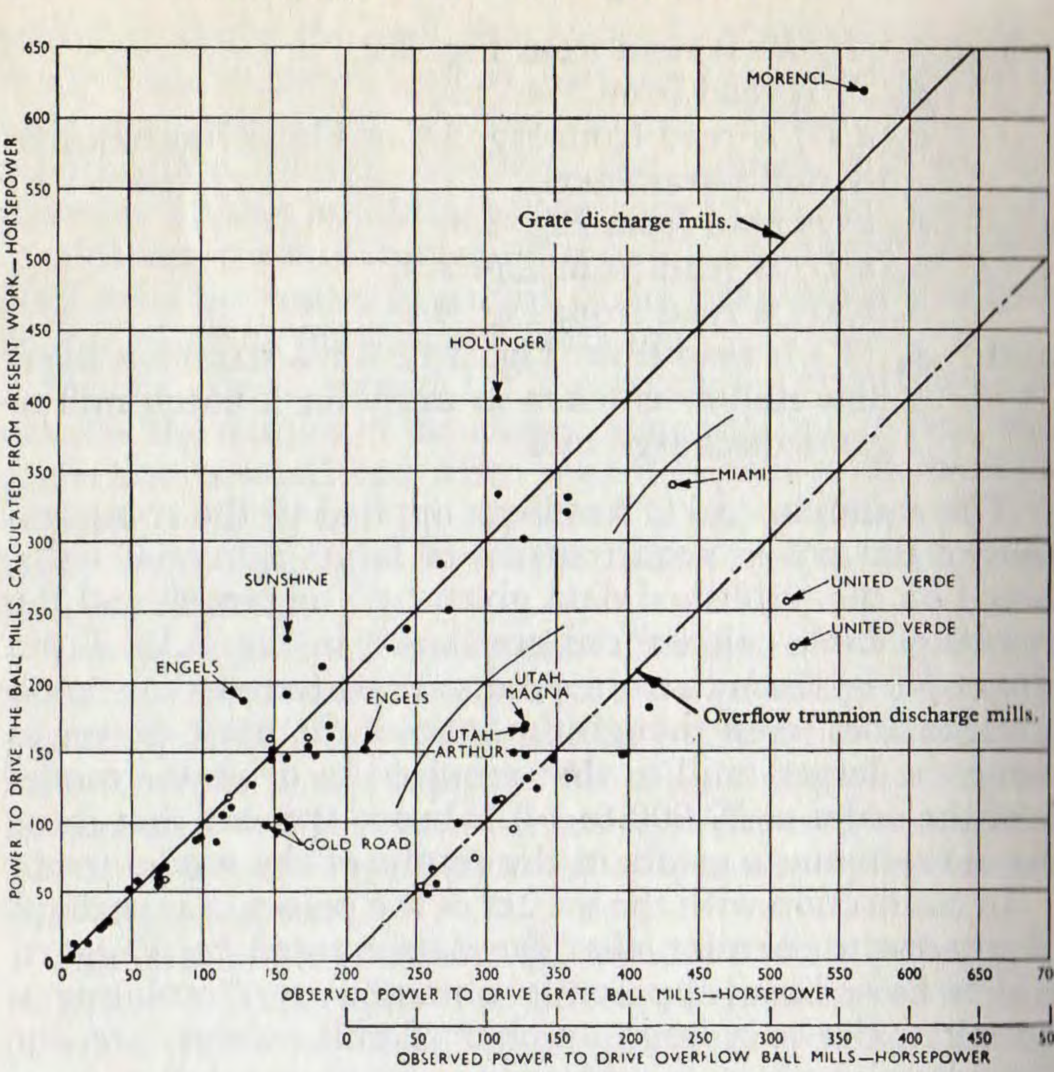


FIG. 3.13

liners. Since the power input, $P$, is related to the diameter $D$ by the expression $P = KD^{2\cdot5}$, the uncertainties in the diameter of the mill can produce considerable inaccuracy in the calculated value of the power input.

(4) The density of the material being ground is not known precisely.

In view of these uncertainties, it is believed that the agreement between the calculated and measured powers is as close as can reasonably be expected, and it may be concluded that the results of this work upon small-scale models may be applied with confidence to the calculation of the power input to industrial mills of even the largest size; that is to machines absorbing even several thousand horse-power. The method of calculation developed in this work will be demonstrated by application to the data, relating to a large mill, published by Carey and Stairmand.(7)

A lifter mill of 6 ft diameter, revolving at 18·7 r.p.m., is grinding coal. The body of the mill is divided into two sections: one section, 7·5 ft long, being charged with 5 tons of steel balls, of 2 in. average diameter, and the other section, 15 ft. long, being charged with 6 tons of "Cylpebs" of $\frac{5}{8}$ in. average diameter.

For the section 7·5 ft long and assuming 40 % voids between the balls and in the coal, the above data lead to:

$$\text{Volume of balls} = 38 \text{ cu. ft}$$

$$\text{and Volume of chamber} = \pi/4 \times 6^2 \times 7{\cdot}5 = 212 \text{ cu. ft}$$

$$\text{Thus} \qquad (J) = 0{\cdot}18$$

From equation (3.5), the critical speed for both sections is 31 r.p.m.

Now from the appropriate graphs,

$$\phi_1'(N_c/N) = 9{\cdot}2, \quad \phi_3(J) = 0{\cdot}65, \quad \phi_4(D/d) = 1{\cdot}0,$$
$$\phi_5(n) = 1{\cdot}0, \quad \text{and} \quad \phi_6(h/D) = 1{\cdot}0$$

and since the particle diameter, $b$, is probably less than about $\frac{1}{8}$ in., $\phi_9(D/b) = 1{\cdot}0$.

Then from the equation

$$\left(\frac{P}{D^5 N^3 \varrho}\right) = \left(1+\frac{0{\cdot}4\sigma}{2\varrho}\right)\cdot\left(\frac{L}{D}\right)\cdot\phi_1'\left(\frac{N_c}{N}\right)\cdot\phi_3(J)\cdot\phi_4\left(\frac{d}{D}\right)$$
$$\times\phi_5(n)\cdot\phi_6\left(\frac{h}{D}\right)\cdot\phi_9\left(\frac{b}{D}\right)$$
$$= \left(1+\frac{0{\cdot}4\times 1{\cdot}4}{7{\cdot}9}\right)\times\left(\frac{7{\cdot}5}{6}\right)\times 9{\cdot}2\times 0{\cdot}605\times 1{\cdot}0$$
$$\times 1{\cdot}0\times 1{\cdot}0$$

or $$P = 820{,}000 \text{ ft pdl/sec}$$
$$= 49{\cdot}5 \text{ h.p.}$$

If, for the section 15 ft long, it is assumed that the "Cylpebs" pack with a voidage of 40%, then $(J) = 0{\cdot}13$ and, by similar calculations

$$P = 66 \text{ h.p.}$$

Thus the power to drive the whole mill

$$= 49{\cdot}5 + 66{\cdot}0 = 115{\cdot}5 \text{ h.p.}$$
$$= 86 \text{ kW}$$

From the published data, the measured power to the motor terminals is 103 kW, and so the power demand of 86 kW by the mill leads to a combined efficiency of motor and transmission of 83%, which is reasonable. Thus the agreement is as close as can be expected.

Consideration of equation (3.11) will show that for speeds less than 80% of the critical,

$$P = D^5 N^3 \varrho . \left(1 + \frac{0{\cdot}4\sigma}{\varrho}\right) . \left(\frac{L}{D}\right) . \left(\frac{N_c}{N}\right)^2 . \phi_3(J) \quad \ldots . \quad (3.12)$$

the remaining groups of the equation being neglected for the time being.

Now, for a given mill filling and a given material being ground, $\phi_3(J)$ and $(1 + 0{\cdot}4\sigma/\varrho)$ are invariant, and for a constant fraction of the critical speed $\phi_1(N_c/N)$ is constant.

Furthermore, since the mill runs at a given fraction of its critical speed, it follows from equation (3.6) that

$$N = K/\sqrt{D}$$

so $$P = D^5 . (K/D)^{3/2}$$
$$= KD^{3{\cdot}5} \quad \ldots . \quad (3.13)$$

This equation is true for complete similarity between length and diameter of the mill but, for a mill of *constant length*, it follows that

$$(L/D) \propto (1/D)$$

and so, in this case,

$$P = KD^{2\cdot 5} \qquad \ldots\ldots \quad (3.14)$$

which is a widely accepted relationship for the power input to a mill.

In fact, it has been found by Gow, Guggenheim, Campbell and Coghill,(8) Fahrenwald and Lee(9) and others that, in practice, the power demand is given more nearly by the relationship

$$P = KD^{2\cdot 6} \qquad \ldots\ldots \quad (3.15)$$

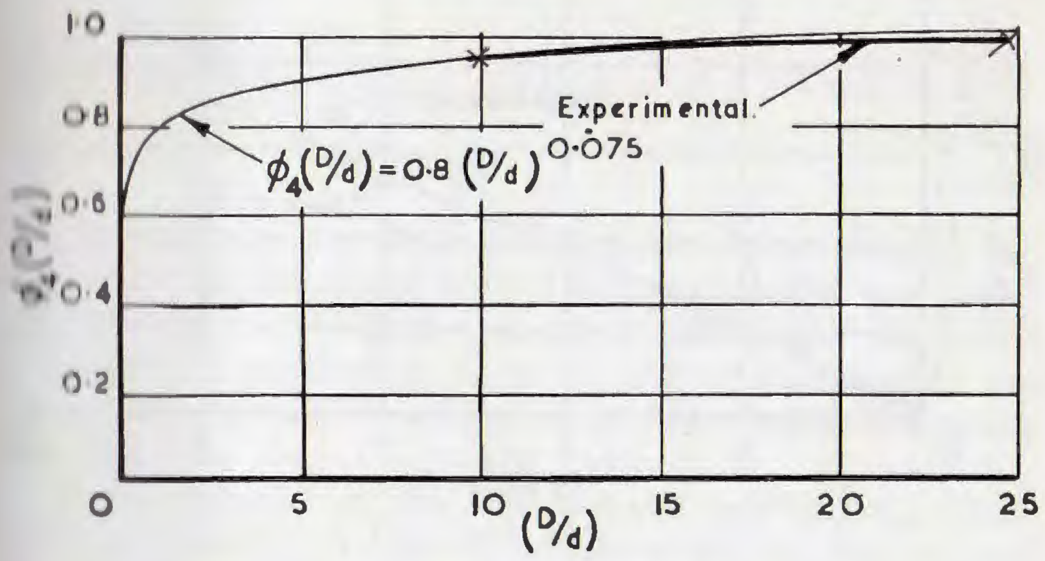


FIG. 3.14

From Fig. 3.7, however, it would be expected that $\phi_4(D/d)$ varies approximately as $(D/d)^{0\cdot 1}$ and in Fig. 3.14 is shown the curve $(D/d)^{0\cdot 075}$ plotted on the same axes as the dotted curve of Fig. 3.7. It is seen that the fit is reasonable.

Thus it follows that in tests in which the *ball diameter is constant* the ratio $(D/d)$ varies directly as $D$, and so the power to the mill, which varies with $\phi_4(D/d)$, will vary as $D^{0\cdot 1}$ approximately. Thus, it follows that, if the ball diameter is maintained constant, the equation (3.14) must be multiplied by $D^{0\cdot 1}$, which brings about agreement with equation (3.15).

One of the earliest attempts to derive, from theoretical considerations, a formula for the power to drive a mill is that of Davis,(10) who starting from the basis of the ball trajectories discussed in the previous chapter, calculated the striking velocity and so the kinetic energy of the elements of the charge in a given trajectory and, by integration, the kinetic energy to the whole charge; this kinetic energy clearly being supplied to the mill from the external source.

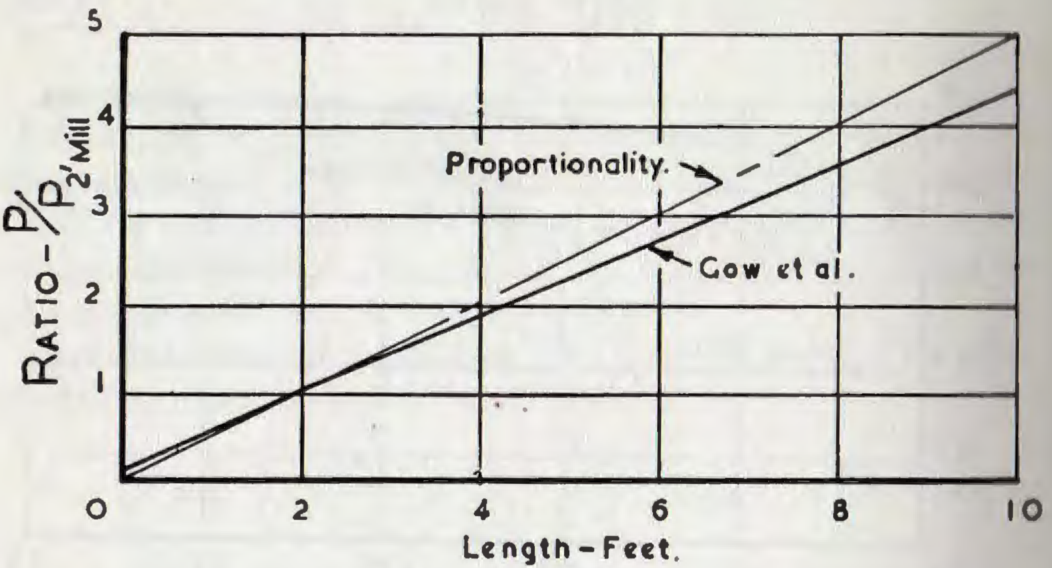


Fig. 3.15

On this basis the following formula for the power input to the shell, when working under conditions of best theoretical efficiency, was obtained:

$$P = WR^{3/2}\left\{0{\cdot}004467\,\frac{(1-K^3)}{(1-K^2)^{1/8}} - 0{\cdot}003700\frac{(1-K^5)}{(1-K^2)^{3/8}} + 0{\cdot}000880\,\frac{(1-K^7)}{(1-K^2)^{5/8}}\right\} \quad \ldots . \quad (3.16)$$

where the value of $K$ is given by the following approximate equation:

$$K = -0{\cdot}024 + 0{\cdot}39(7-10J)^{1/2} \quad \ldots . \quad (3.17)$$

The equation (3.16) is not directly comparable with equation (3.11), since for a given composition of charge, the weight of the charge, $W$, cannot be varied independently of the diameter of the mill and similarity maintained at the same time. Thus, the functional relationship $\phi_1(J)$ of the previous work is absorbed in the form of equation (3.17). Furthermore, the equation of Davis is derived for the conditions of maximum efficiency and so is not general.

An equation developed by Bond⁽¹¹⁾ for mills dry-grinding is

$$P = 1{\cdot}26\left\{D^{0\cdot4} \times \frac{N}{N_c}(6{\cdot}16-5{\cdot}75J) - 0{\cdot}1 \times 2^{10(N_c/N-0\cdot6)-1}\right\} \quad \ldots\ldots (3.18)$$

where $P$ is the power draft in kilowatts per ton of grinding balls. The equations of both Davis and Bond are based only on the mill diameter, the mill filling and the speed of rotation expressed in terms of the critical speed; the forms of the equations arising from the empirical expressions defining the relationships between several variables. Since the weight of the ball charge enters both these equations directly, it follows that the power is proportional to the length of the mill; a conclusion which is in accord with the equation of Rose and Evans.(12)

It has been suggested by Gow, Guggenheim, Campbell and Coghill,(13) however, that the power does not vary directly as the length of the mill, but these workers give an expression which reduces to

$$\phi_2(L/D) = (0{\cdot}5L-1)K+1 \quad \ldots\ldots (3.19)$$

where $L$ is the length of the mill and $K$ is a constant having the value of 0·9 for mills of length less than 5 ft and 0·85 for mills of length greater than 5 ft. This function is plotted in Fig. 3.15, and it is seen that for long mills the power is considerably smaller than is demanded by a linear function.

The higher power requirements of a short mill would, perhaps, be expected, since the end plates of the mill might cause a "piling-up" of the charge at the ends of the mill. Such an effect would, however, be expected to be negligible in a long mill. Thus, this deviation for a long mill is surprising and it is believed that this formula should be accepted with caution.

Hancock(14) proposed the formula

$$P = 0{\cdot}00078C^3Ln\delta \qquad \ldots. \quad (3.20)$$

where $C$ is the length in feet of the chord defined by the surface of the mill charge, $L$ is the length of the mill in feet, $\delta$ is the mean specific gravity of the charge and $n$ is the speed of rotation in r.p.m. This formula approximates to the formula of Rose and Evans insofar that the term $\varrho(1+0{\cdot}4\sigma/\varrho)$ is proportional to the mean density of the pulp, and so to $\delta$ of equation (3.20), and in that the power is proportional to the length of the mill.

Furthermore, the quantity $C^3$ may be written:

$$C^3 = D^3(C/D)^3$$

but $(C/D)$ is the sine of the semi-angle, $\Delta$, subtended by the surface of the charge at the centre of the mill and so

$$C^3 = D^3 \sin^3 \Delta$$

Examination of Fig. 3.16 shows, however, that for the practically useful range, of up to 50% filling, the curves of $\sin^3 \Delta$ and $\phi_3(J)$ practically coincide. Thus

$$C^3 = D^3 \times \phi_3(J)$$

Also from equation (3.6) it follows that for constancy in the ratio of running speed to critical speed

$$n \propto 1/\sqrt{D}$$

Then, on making these substitutions, equation (3.11)

$$P = D^5N^3\varrho \times \phi_1'\left(\frac{N_c}{N}\right) \times \phi_3(J) \times \left(1+0{\cdot}4\frac{\sigma}{\varrho}\right) \times \left(\frac{L}{D}\right)$$

becomes

$$\begin{aligned} P &= K_1 D^5 N^3 \times \frac{g}{DN^2} \times \phi_2(J) \times \frac{L}{D} \times \varrho\left(1 + 0{\cdot}4\frac{\sigma}{\varrho}\right) \\ &= K_2 N \times \{D^3 \times \phi_3(J)\} \times L \times \delta \\ &= 0{\cdot}00051 NC^3 L\delta \qquad \ldots . \quad (3.21) \end{aligned}$$

Thus, the general form of the two equations is in close agreement but the value of the coefficient (0·00078) appears to be in error.

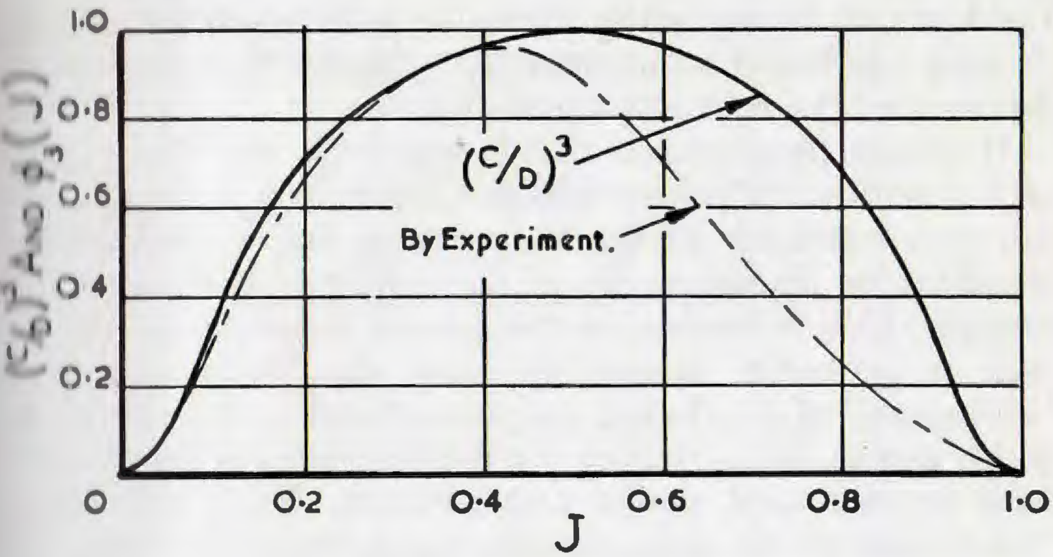


FIG. 3.16

For values of mill filling greater than 50%, however, the agreement is not good but, in practice, high mill fillings are perhaps not so frequently used as the lower values so it may be concluded that, provided that the correct value is assigned to the coefficient, the equation of Hancock, in spite of its simpler form, is, over a practical range of variables, in good agreement with those of other workers. Thus this formula includes, approximately, the dimensionless group $\phi_3(J)$ of the previous work.

The dependence of the power demand of a mill on the nature of the pulp does not appear to have received a great deal of study, and, in general, observations on this matter take the form of general statements. For example, Taggart(15) states that, other things being equal, wet milling requires 60–90 % of the power for dry milling. This view is confirmed by calculations of the present authors, based on figures for two mills of about 8 ft diameter and 45 ft long kindly made available by Messrs. Edgar Allen & Co. In this case the power as calculated by the formulae of Rose and Evans was in extremely good agreement with the observed power requirements of the mill when dry grinding (within 5 %). However, the power calculated on the basis of dry-grinding formulae with modified charge density was found to be about 20 % higher than the actual demand of the mill when grinding wet.

It would be expected that a fine "smooth" fluid pulp would reduce the power demand, since such a pulp would act as a lubricant to the mill charge and so reduce the equilibrium displacement of the centre of gravity of the charge. This reduction of the power requirement of the mill is probably bound up with the phenomenon of "slumping" of the charge, mentioned earlier. More viscous pulps would, by increasing the displacement of the charge, lead to increased power consumption. With increasing "tackiness" of the pulp, a reduction of the power requirements would be expected, since, in such a case the balls might adhere so well to the shell that a part of the charge is spread over the shell and, thus, is balanced about the axis of rotation. This decrease in power demand with pulps of increasing thickness is in accord with the observations of Gow, Guggenheim, Coghill and Campbell,(16) but these workers did not carry the tests to the point where the balls adhered to the shell of the mill. This effect is, however, also observed in mills which are used for the dispersion of pigments in varnish, and similar duties, and so is very real.

From what has already been said about the interaction between the coefficient of friction and the magnitude of the ball charge, it would be expected that the effect of the change in pulp conditions would be relatively more important in mills for which the charge is small. This conclusion in fact follows from the observations of Gow, Guggenheim, Campbell and Coghill(16). Thus it would be expected that, for large fillings, the effects of the variation of the pulp characteristics would be, largely, in accordance with what would be expected to result from the consequential changes in the coefficient of friction involved.

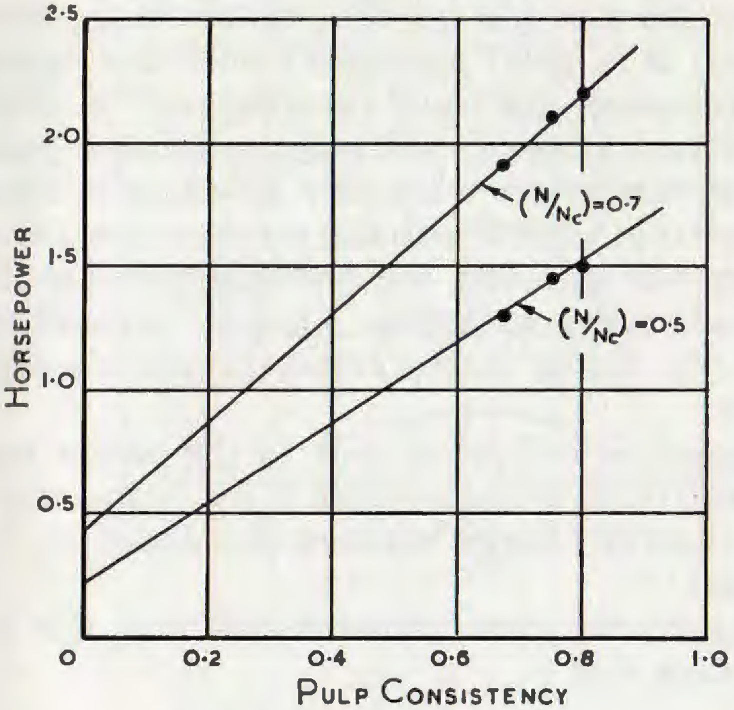


FIG. 3.17

In view of the lack of data on this subject, it would appear that the best corrections for pulp characteristics which can be made is to assume that, for very thin pulps and moderate values of $J$ and $(D/d)$ the power required is 60 % of that for dry milling, and then to use a linear interpolation for less liquid pulps. For larger mill fillings or greater values of $(D/d)$ the reduction in power would be less than for the previous case and the multiplication factor should be modified accordingly. The general correctness of this view is supported by Fig. 3.17 in which the results of tests by Gow, Guggenheim, Campbell and Coghill,(16) are plotted. Although these points are small in number and rather closely spaced they give lines which when extrapolated to zero pulp density give 0·22 and 0·45 horse-power as the powers to drive the mills without pulp charge; these figures being in the ratio 2·05 to 1·0.

Reference to equation (3.9), however, shows that, all other variables being maintained constant, the ratio of the powers is proportional to the ratio of the square of the speed; in this case $(0{\cdot}70/0{\cdot}50)^2$, that is in the ratio 1·96 to 1·0, which is in good agreement with the figure 2·05 to 1·0. Furthermore, for a mill running at 70 % of the critical speed the ratio of the power when grinding a pulp of 50 % density to the power when dry grinding is about 0·6 to 1·0: which is in accord with the previous general statement that when wet grinding the power demand is about 60 % of that when dry grinding. Thus it appears that, very roughly, the linear interpolation previously suggested is adequate.

The possibility of calculation of the power required to drive a mill from the knowledge of the horizontal displacement of the mill charge has been developed by Carey and Stairmand.(17)

From the principles discussed earlier in this work, it is easily shown that

$$P = WNH_h(1/3{\cdot}14) \qquad \ldots . \quad (3.22)$$

where $P$ is the power input to the shell in kilowatts, to maintain the displacement of the charge, $W$ is the weight, in tons, of the grinding media plus powder, $N$ is the mill speed in r.p.m. and $H_h$ is the horizontal displacement in feet of the centre of gravity of the charge. Thus, the power input may be calculated provided the value of $H_h$ can be obtained.

For this purpose Carey and Stairmand furnish a graph which gives the value of the ratio $H_h/D$ ($D$ is the mill diameter), in terms of the mill filling and the ratio of the speed of rotation of the mill to the critical speed; this graph being

$\bar{W} = (\pi D^2/4)\, \rho_{\phi} \cdot L J$, $P = 0{,}3185\, \bar{W} N H_h$ kW shown in Fig. 3.18. These curves are stated to be applicable to mills having diameters ranging from a few inches to several feet and, in practice, to give reasonably reliable results.

Analysis of this method in the light of the previous discussion shows that the effects of the weight of the charge, and so $\phi(1+0{\cdot}4\sigma/\varrho)$, are included. Also the effects of $\phi_1'(N/N_c)$, $\phi_3(J)$ and of lifters are included, and so it appears that only variables which have been shown to be of comparatively minor importance are omitted. Thus, it would be expected that this treatment would have an accuracy sufficient for most practical purposes.

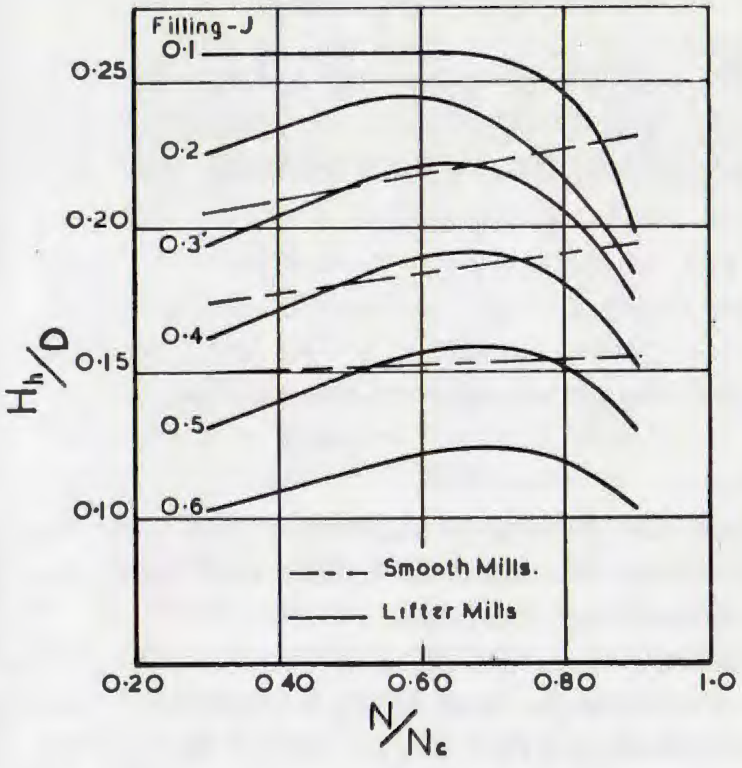


Fig. 3.18

The power required to drive a rod mill does not appear to have been extensively studied. As a first approximation, it would be expected that the value of the function $\phi_{10}(\sigma/\varrho)$ would be different to that for a ball mill but that the values of the remaining functions would not be seriously altered.

Since the "voidage" in a bed of circular rods, packed in the tightest array, is about 10 % (0·095) and assuming the voidage of the powder to be 40 %, it follows that

$$\phi_{10}\left(\frac{\sigma}{\varrho}\right) = \frac{(1-0{\cdot}1)\varrho+0{\cdot}1(1-0{\cdot}4)\sigma}{(1-0{\cdot}1)\varrho}$$
$$= 1+0{\cdot}067\sigma/\varrho \qquad \ldots . \quad (3.23)$$

Also, for a given value of mill filling ($J$) the load of rods in the mill will exceed that of balls in the ratio of the volume of rods to the volume of balls, that is,

$$\frac{1-0{\cdot}1}{1-0{\cdot}4} = 1{\cdot}5$$

Thus, the power to drive a rod mill, without a powder charge, will be given approximately by

$$P_R = 1{\cdot}5P_B \qquad \ldots . \quad (3.24)$$

where $P_R$ is the power to drive the rod mill and $P_B$ is the power to drive the corresponding ball mill, as calculated from the foregoing formulae.

It is probable, however, that the action of the powder to dilate the charge, and so to reduce the horizontal displacement of the centre of gravity of the charge, would be more marked with rods than with balls. This would lead to a reduction in the actual power to below that which would be calculated by the use of the formulae of the ball mill, as modified by the use of equations (3.23) or (3.24). This effect would, however, probably be small and so could be neglected.

A further case which requires consideration is that in which the grinding media is of some form such as "Cylpebs"; off-cuts from scrap drill rods sometimes being used in the mining industry. Also in some cases, though rather rarely, large pieces of ore are used as a grinding medium. It appears, however, that only in the case of extremely high costs for the carriage of replacement balls can the use of such lumps of ore be justified on economic grounds; since the grinding efficiency is adversely affected thereby and the milling cost correspondingly increased.

The use of such non-circular or non-spherical materials would lead to a "locking" of the elements of the charge, and so to an increased displacement of the centre of gravity of the charge, with a relatively increased power demand. The power demand would, furthermore be modified by the tightness of packing of the grinding bodies, which is different to that for balls, and by the density of the material. The power to drive mills charged with such materials does not appear to have been studied, however, and, with the present state of knowledge, little more than rough estimates of the power calculated on the basis of the previous equations modified on the lines suggested, can be made.

The foregoing work deals entirely with the power necessary to keep the charge within the mill shell in motion. Clearly, in order to determine the power to be supplied by the driving motor, it is necessary to add to this figure the power lost in the bearings supporting the mill shell, in the bearings supporting the intermediate speed shafts and in the reduction gearing. The estimation of these losses, is, however, a matter of general engineering knowledge, and is not specific to the tumbling mill, and such matters are specifically excluded from the present work.

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CHAPTER 4

# ON THE COMMINUTION OF SOLID BODIES

The problems associated with comminution of solid bodies are numerous and complex. In spite of the large amount of investigation which has been undertaken, the present state of knowledge is such that there is no theory by means of which the behaviour during fracture of even a single particle under the simplest possible mode of loading can be predicted in reasonable detail or with moderate accuracy.

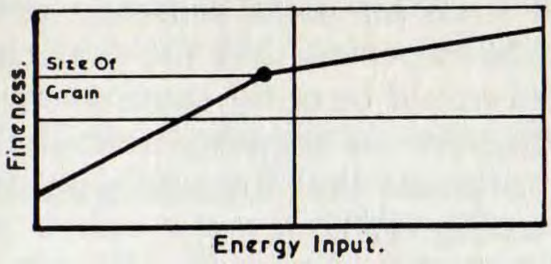
In the absence of an adequate theory applicable to even a simple case, it is clearly unreasonable to hope that the behaviour of a comparatively complicated system, such as a mill, in which numerous particles are involved and the mode of loading gives rise to both impact and abrasive grinding of the particle, can be predicted from theoretical considerations. Because of this absence of adequate theoretical knowledge much of the design of large industrial mills is based on previous experience with similar mills or is deduced from tests on pilot plant. In spite of the absence of adequate theories, it is intended to give in this chapter a brief survey of the present knowledge of the subject of the fracture of solid particles, since it is believed that such knowledge is desirable if the problems of grinding are to be understood.

The materials which are ground industrially may be either homogeneous or heterogeneous. Thus, graphite, molybdenum disulphide, pigments and drugs are, when technically pure, homogeneous from the standpoint of uniformity of crystalline make-up. A particular crystalline material will tend to exhibit characteristic mechanical properties which spring from the crystal structure and associated factors, such as crystallite size and arrangement of the crystallites within the macro crystal or powder particle. Such materials are homogeneous.

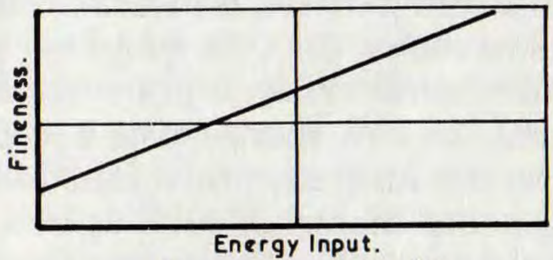
Materials like iron ore, however, will generally be composed of particles of almost pure iron oxide, intermixed with particles of silica. Thus, metal ores in general, coal and cement clinker are heterogeneous materials. For reasons which will be discussed later, there is sound evidence that these two types of material have different fracture characteristics. The line of demarcation between the two types of material is not sharp, however, for although a heterogeneous material is fairly obvious, homogeneous materials have internal defects which introduce some of the characteristics of heterogeneous materials.

Consider, in the first place, the facture of a heterogeneous particle; for example, a cube of granite or iron ore. Since, in general, the cementing medium at the grain boundaries in such a material may be considered to have a lower strength than the grains themselves the fracture will, in the main, occur by failure of the grain boundaries. After a while the particle will be broken down into the constituent grains. When this stage has been reached further size reduction can only be brought about by fracture of the grains themselves. Since however, as stated, it is probable that the crystals of such materials are stronger than the material at the boundary, grinding beyond this point is more difficult than in the earlier stage. In particular, Hüttig(1) has applied these considerations to the grinding process.

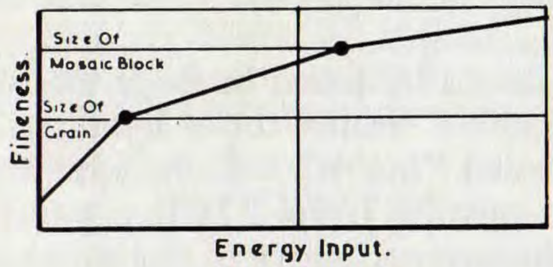
Thus, a graph of fineness attained against the expenditure of work necessary to attain the fineness would be expected to have a form somewhat as shown in Fig. 4.1a. Clearly, in the case of metals, it would be highly improbable that all the grain boundaries would fail before any of the crystals failed. Micro-photographs, by Felix and Geiger,(2) of samples of steel which have failed under brittle conditions show clearly that the fracture is propagated across crystals in which the planes of slip are unfavourably situated with regard to the forced applied. In such cases the stresses cannot be reduced by plastic deformation and so the cohesive strength is exceeded in the cleavage plane. For most materials therefore, failure of some crystals would occur simultaneously with the failure of the boundary and this would eliminate the sharp corner from the curve and so give rise to a non-linear relationship. In the crushing of a homogeneous material, however, the case would be different. If the crystal lattice were perfect and the platens of the press and the polished faces of the



Heterogeneous Material.
a.



Homogeneous Material.
b.



Real Crystal.
c.

FIG. 4.1

crystal were absolutely plane and parallel, then theoretically the stress would be uniform throughout the particle. Presumably, when the stress at which failure occurred were reached, the crystal would disintegrate into a dust in which the particles would be of the same order of size as are the crystallite or elementary blocks of the crystal lattice. Even if such an ideal crushing process did not occur, it would be expected that the particles of a homogeneous material would be of the same strength, regardless of size, and so the curve of fineness attained plotted against work required to attain that fineness would be a straight line, as shown in Fig. 4.1b.

That a crystal of a homogeneous solid will tend to break down on comminution and reproduce the characteristic geometry of the larger particle, is indicated by the work of Stott,(3) which has shown that, on grinding, graphite, talc and other laminar solids retain a plate-like form down to the smallest sizes. In this there exists a condition intermediate between the ideal case mentioned above and the case of heterogeneous materials such as iron ore or other materials of mixed crystalline make-up. The fact remains, however, that no real crystal is perfect, and so the crushing of a truly homogeneous crystal is of theoretical interest only.

The imperfections in a real homogeneous particle may be roughly classified under three headings. In the first class may be placed "micro" defects, which may be of the form of an ion missing from a lattice point or a similar defect in the ultimate structure of the material. Defects of this nature are probably so small in relation to even the smallest industrial particle, and so uniformly distributed throughout the particle, that, although they reduce the strength of the material below that which the same material would have if the lattice were perfect, they have no marked effect upon the fracture characteristics of the material; which remains, in this respect, homogeneous.

In the second class may be placed "macro" defects. Defects of this nature may be flaws or cracks extending across an appreciable fraction of the cross-sectional area of the particle. They may arise from defects in the growth of the crystal, thermal stresses, shock loading, etc. Defects of this nature, if present in any appreciable number, are of importance in connection with fracture since, clearly, they constitute planes of weakness, similar to the grain boundaries in a heterogeneous particle, and so would give rise to a "grinding characteristic" curve as for a heterogeneous material, Fig. 4.1a.

In the third class may be placed what, for brevity, may be called "mosaic" defects. The basic idea underlying this form of defect is that there is evidence that materials are built up of elementary "bricks" having a size of the order of a hundred lattice dimensions. Thus, for example, Riley(4) suggests that the micelle, which represents the elementary unit in coal and coke, has a size of about 35–40 Å; Humphreys-Owen(5) states that mineral salt (NaCl) has an elementary structure of about 1500 Å. One of the present writers, Rose,(6) has shown that curves of both the bulk density and reflectivity of graphite have a discontinuity, which probably indicates a change of structure, or a change in the surface condition of the primary particles forming the aggregate, at a particle size of about 500 Å.

If now the crystal structure is "perfect", the elementary bricks are arranged in an ordered way; the adjective "perfect" merely indicating that the array of the "bricks" is ordered and *not* that there are no defects, of the type classified as "micro" defects above, in the crystal lattice. If, however, the crystal is "mosaic", the elementary blocks are not in an ordered array and such a crystal would be expected to be weaker in brittle fracture than the corresponding crystal with ordered array. A crystal structure of this nature would again suggest that grinding would become more difficult when the particle size reached that of the mosaic block. Starting from a large particle size, the grinding characteristic curve for a mosaic crystal would be expected to be in three parts, as shown in Fig. 4.1c. In practice, the grinding of materials to below the mosaic block size is rarely carried out and so evidence as to the behaviour of particles in this size range is lacking.

It is known that, as grinding proceeds, there is an increasing degree of aggregation of the particles, which makes it increasingly difficult to separate the primary particles of the aggregate as, for example, in the preparation of dispersions of the solid material in a liquid vehicle. The underlying reasons for this are not clearly understood, but a factor is probably the increase in the magnitude of the surface forces as the particle size is reduced, these forces approximating to those which hold the material in solid form as the particle size approaches that of the elementary units from which the material is constructed.

The foregoing elementary discussion is relevant only to the crushing of a single particle but in practice a number of particles, often in a bed of several particles thickness, are crushed and so it is probable that the behaviour during multiple-particle crushing will differ widely from that outlined. In addition, the foregoing discussion involves only the energy supplied to the particle and not that supplied to the crushing device. This difference is of considerable importance, however, because the results obtained from certain types of crushing are sometimes interpreted as arising from the crushing characteristics of the material. As an example of this fallacy may be cited the use of the results of tests carried out upon a ball mill. From Chapter 3, it is easily seen that the change in particle size has practically no effect upon the power to drive a ball mill and so the energy input to the mill is proportional to the time of milling, as in Fig. 4.2. When, however, the mill contents is sampled and the fineness of the product plotted against the time of grinding, the result is a curve, as shown.

At first sight this curvature would appear to, and probably does in part, arise from the increased strength of the fine particles. In fact, however, many other more probable mechanisms which would lead to this result are operative; for example the reduced area around the contact point between a pair of balls, within which crushing must take place, will lead to a decreased rate of grinding as the fineness increases. Another possibility is the carrying out of a fine particle by the air expelled when the falling ball approaches the stationary one, or even welding together of the finest particles by the blow would tend to produce a line of the type shown. Thus, there is certainly no ground for stating, without further evidence, that the curvature of the line constitutes proof of increasing strength of the particles with decreasing size. Again, the existence of fine material, along with the coarser particles, may exert a cushioning effect during milling; the fine material interfering with the mechanical action of the ball on the crystalline material being ground by forming a resilient matrix in which the larger particles become embedded.* That this mechanism is of importance is supported by the observation that the rate of grinding in a batch mill is increased when the charge is removed from the mill, the fines sieved out, and the oversize material remaining again returned to the mill.

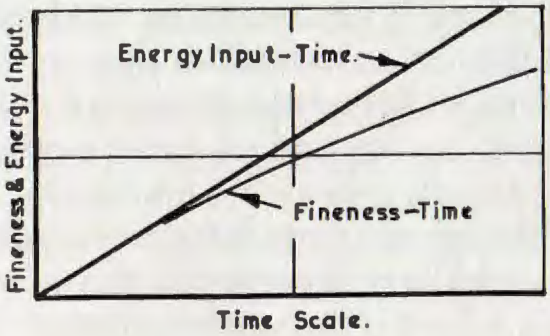


FIG. 4.2

Either of these explanations forms a more practical explanation for the decrease in the rate of size-reduction of a powder with increased milling time than does that based on the hypothesis that the mechanical strength of the material increases with a decrease in particle size.

At this point it is interesting to consider the distribution of the energy supplied to a particle during crushing.

This energy would be distributed in:

*To embed =

(1) Elastic deformation of the particle.
(2) Plastic deformation of the particle.
(3) Gliding or slip on atomic planes, twinning and possibly other lattice re-arrangements within the mineral.
(4) Increased surface energy of the body.

The energy corresponding to permanent lattice movements, which will be small for hard, brittle materials, and to increased surface energy, is absorbed during the crushing process and does not reappear as heat. The remainder of the energy supplied, however, will reappear as heat and this difference at once gives rise to the idea of the energy which is "necessary" for crushing and that which is "sufficient". Thus, it is essential that sufficient energy should be supplied to bring about enough elastic deformation for fracture to occur, and no less quantity will suffice. When fracture occurs, however, this energy, less that corresponding to the permanent lattice rearrangements and to the increased surface energy, reappears as heat and it is this difference which is measured when a mill is run in a calorimeter. Since, however, the energy for elastic, and possibly that for a part of the plastic, deformation is essential if fracture is to take place, the use of the absorbed energy as a standard of comparison is grossly misleading as to the efficiency of a crushing process. As an example of this discrepancy, it has been observed that the energy supplied in free crushing is about five times that calculated on the basis of surface energy, whilst for a ball mill the value of the ratio is often between 500 and 5000.

When a bed of particles is crushed the energy expended in friction between the particles must be supplied, but, again, this ultimately reappears as heat and performs no useful work in grinding. Thus, friction in the bed of particles will, again, lower the apparent "efficiency" of a crushing process.

Turning now to the so-called "laws of crushing" which have been propounded, the earliest of these is that of Rittinger,(7) who in 1867 stated, apparently axiomatically, that "the energy necessary for crushing is directly proportional to the new surface produced". Continual reference to this "law" is found in the literature so it is important that its implications should be carefully considered.

Purely from the point of view of an energy balance, this law would hold provided:

(1) The whole of the energy considered is absorbed as surface energy.
(2) The energy per unit surface is independent of particle size.

The law could not hold when strain energy and plastic deformation energy is included. It cannot therefore be expected to apply to the *whole* comminution process or to the comminution of ductile materials. It would be expected to be applicable to the comminution of brittle materials when the energy expended in elastic deformation is deducted from the energy input to the particle.

Since, for spherical particles the specific surface, in area units per unit mass, is given by

$$S = 6/\sigma d \qquad \ldots. \quad (4.1)$$

where $\sigma$ is the density of the material of the particle and $d$ the particle diameter, it follows that the increase in specific surface during grinding is given by

$$\begin{aligned} S_1 - S_0 &= (6/\sigma)(1/d_1 - 1/d_0) \qquad \ldots. \quad (4.2) \\ &= S_0(R-1) \qquad \ldots. \quad (4.2a) \end{aligned}$$

where $R$ $(= d_0/d_1)$ is the reduction ratio.

If now $W$ is the energy necessary for the creation of one unit of new surface the ratios of the energies necessary to reduce unit mass of a given material through the reduction ratios $R_1$ and $R_2$ is given by

$$\frac{W_{R_1}}{W_{R_2}} = \frac{R_1 - 1}{R_2 - 1} \qquad \ldots. \quad (4.3)$$

and if the reduction ratios are so large, in comparison with unity, that it may be neglected, then

$$W_{R_1}/W_{R_2} = R_1/R_2 \qquad \ldots . \ (4.3a)$$

Furthermore, the energy necessary to reduce unit mass of material from size $d_0$ to size $d_1$ is given by

$$W_t = (6/\sigma)W(1/d_0 - 1/d_1) \qquad \ldots . \ (4.4)$$

$$= W(S_0 - S_1) \qquad \ldots . \ (4.4a)$$

These expressions, the use of which involves a knowledge of the results of sieve analyses or of the specific surface determination, are easily applied and are widely used for the control of fine grinding operations.

The law of Rittinger appears to have remained unchallenged until 1885 when Kick⁽⁸⁾ propounded the "Gesetz der proportionalen Widerstände", but since that time a controversy has raged, concerning the validity of the two laws, which has not yet been resolved.

The "law" as formulated by Kick is:

(1) The quantities of work necessary in order to produce equivalent changes in shape in two geometrically similar bodies of the same material are in the ratio of the volumes or the weights of the bodies.

(2) Bodies of given shape or form, in order to undergo a definite given subdivision or change in shape, require an expenditure of work equal to the product of the weight and a quantity which is the quantity of work necessary for unit weight of the same shape to undergo the same deformation or subdivision.

(3) The forces necessary for corresponding changes in shape of two geometrically similar bodies of the same material are in the ratio of the corresponding cross-sections or the surfaces of the bodies.

Actually the second statement has precisely the same information content as the first but is stated in an alternative form.

[illegible]

Kick obtained experimental verification of this law from tests on the crushing of simple objects and, by applying the law to size reduction generally, was apparently able to refute the law of Rittinger.

As in the case for Rittinger's law, an expression for the work input in order to bring about any specified reduction ratio may be deduced. Thus, Kick's law states that the work necessary to reduce a body of unit size to a number of particles of, say, half the size is constant. Each of these particles, however, requires only one-eighth ($\frac{1}{2}^3$) of the energy of the original body to reduce *their* size to one-half, but, as there are now 8 ($2^3$) of them, the energy to reduce the eight to one-half their size is equal to that required to reduce the original particle to particles of one-half of the size of the original body. Thus, to reduce *a unit mass* of a material to a given fraction of the original size requires the expenditure of a constant amount of energy; regardless of the initial size of particle.

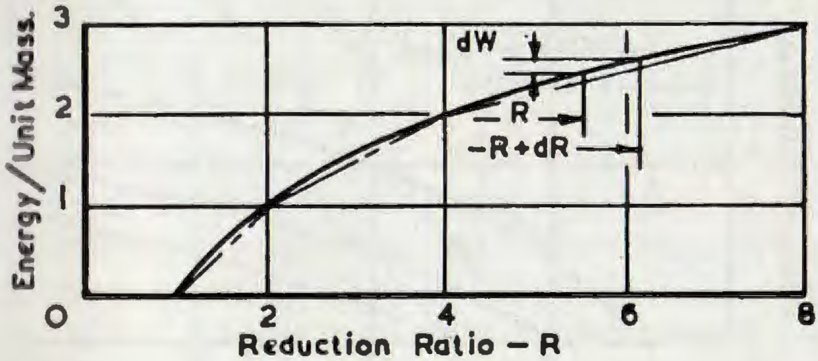


FIG. 4.3

This can be illustrated graphically as in Fig. 4.3. Clearly, if the reduction ratio $R$ is defined as $d_0/d_1$, when the material has been reduced to one-half the original size, $R=2$ and one unit of energy has been supplied. To reduce the size to one-half again changes the reduction ratio from 2 to 4, but again one unit of energy is required; and so on for further subdivision.

The relationship between $W$ and $R$ for such a case is clearly

$$W = K \log R \qquad \ldots\ldots \quad (4.5)$$

but this relationship can be obtained formally as follows:

$$\begin{aligned}(1/K)\,\mathrm{d}W &= \text{Increase } \mathrm{d}R \text{ in the reduction ratio expressed as a fraction of the initial value of } R\\ &= (R+\mathrm{d}R-R)/R\\ &= \mathrm{d}R/R\end{aligned}$$

or, on integration,

$$W = K \log R \qquad \ldots\ldots \quad (4.6)$$

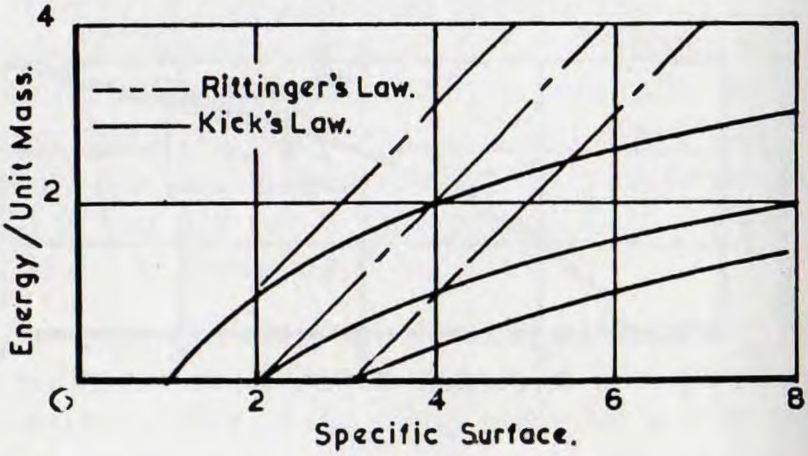


FIG. 4.4

Alternatively, since $R = d_0/d_1$ or $R = S_0/S_1$, equation (4.6) may be written:

$$W = K \log (d_0/d_1) \qquad \ldots\ldots \quad (4.6a)$$

or

$$W = K \log (S_1/S_0) \qquad \ldots\ldots \quad (4.6b)$$

The essential difference between the law of Rittinger and that of Kick is that according to Rittinger the energy required to crush a particle depends upon the initial size of the particle, whereas the law of Kick states that the energy required depends only upon the reduction ratio. The relationships between energy input and total surface, for various values of initial fineness, according to the laws of Rittinger and Kick are shown in Fig. 4.4.

The foregoing analysis brings out a number of interesting points in connection with the fields of applicability of these two laws. In the first place Kick's law states that the energy to bring about a specified change in the size of a unit mass of material is constant and, furthermore, since the strain energy to induce a certain critical stress in a unit mass of material is also constant, it would appear that the forces involved in breakage in accordance with Kick's law are body forces and, furthermore, after fracture the energy should appear as heat. Thus, Kick's law would be expected to hold for particles for which the surface energy is negligible; that is for large particles during coarse crushing.

Conversely, Rittinger's law, which states that the energy required for fracture is proportional to the new surface formed, would be expected to hold for very fine grinding since, in this case, the surface is large and the surface energy requirements could swamp the elastic energy requirements. In fact there is evidence that, in practice, coarse crushing is roughly in agreement with Kick's law, whereas fine grinding is more nearly in accord with Rittinger's law. At this point the implication of these two laws in relation to the physical process of crushing will be considered.

Smekal(9) has pointed out that although Rittinger's law represents a logical interpretation of the process of formation of new surface, the manner in which the process is initiated is disregarded. In technical size reduction, the particles are crushed by impact, pressure or abrasion. The surface energy of the resulting newly formed surface is gained at the expense of the elastic deformation energy of the body; the balance of the energy reappearing as sound and heat. Smekal has also pointed out that particles cannot be considered as being of homogeneous structure, and Hönig(10) has shown clearly how the influence of the microstructure affects the validity of the laws of crushing. The importance of inhomogeneity in the particle is shown by the fact that it is generally accepted that the calculated chemical bond strength of a material is of the order of 100–1000 times the actual mechanical strength of the material. Furthermore it is generally believed that the discrepancy is due to points of inhomogeneity in the crystal structure; the elastic energy for fracture being concentrated at these points due to notch effects.

*υπονοούμενον It is implicit* in the Rittinger law of crushing that the particles are completely homogeneous, and in the Kick law of crushing that the particles are either homogeneous, or that the internal structure of externally geometrically similar particles is also geometrically similar; that is, the distribution of faults in the internal structure is independent of the volume of the particle. This latter assumption is clearly, in a real particle, of doubtful validity. Both the laws of crushing discussed therefore contain assumptions implicit in their statement which recent theoretical and experimental work on the fracture of particles has shown to be of doubtful validity. In fact, the process of size reduction has been shown by Smekal,⁽⁹⁾ Hönig,⁽¹⁰⁾ Axelson and Piret,⁽¹¹⁾ and others to be so complex that the existence of any general rigid law of crushing is doubtful. Nevertheless, these laws have been established over the last fifty years, as being useful tools for the predicting of the performance of size-reduction machinery.

Within the last few years, a new theory of comminution has been developed by Bond.⁽¹⁷⁾ It is based on the analysis of actual size-reduction machinery rather than on theoretical consideration of fracture. According to this theory, termed by Bond the "Third Theory of Comminution", the effective work done in crushing and grinding results primarily in the formation of crack tips and it is directly proportional to the total crack length formed. In equidimensioned particles the total crack length is stated to be equivalent to the square root of one-half the surface area of the particle. The total work input represented by a given weight of a crushed or ground product is therefore inversely proportional to the square root of the diameter of the particles of the product.

If $W$ represents the work required to reduce, from a feed with 80 % passing $F$ microns to a product with 80 % passing $P$ microns a unit weight of material, then

$$W_i = \frac{W\sqrt{F}}{\sqrt{F}-\sqrt{P}} \cdot \frac{\sqrt{P}}{100} \qquad \ldots. \; (4.7)$$

where $W_i$ is the "work index" or the kilowatt-hours per ton required to reduce from a theoretically infinite particle size to 80 % passing 100 microns.

Equation (4.7) can be transposed so that when the work index $W_i$ is known the work input $W$ required by any feed size $F$ and product $P$ can be found. Thus,

$$W = W_i \frac{\sqrt{F}-\sqrt{P}}{\sqrt{F}} \cdot \frac{100}{\sqrt{P}} \qquad \ldots. \; (4.8)$$

If $R$ represents the reduction ratio, or $F/P$, then

$$\frac{\sqrt{F}}{\sqrt{F}-\sqrt{P}} = \frac{\sqrt{R}}{\sqrt{R}-1}$$

and

$$\frac{W_i}{W} = \frac{\sqrt{R}}{\sqrt{R}-1} \cdot \frac{\sqrt{P}}{100} \qquad \ldots. \; (4.9)$$

The mesh size, in microns, of the sieve through which 80 % of the material passes is selected as the criterion of feed and product sizes since this is a figure which is readily obtainable by plotting the results of screen analyses. It is claimed to be more reproducible than the size of mesh through which 100 % of the sample passes, but other workers use the sieve which passes 100 % of the sample with fair success.

It has been pointed out by Walker, Lewis, McAdams and Gilliland(13) and more recently by Coulson and Richardson(14) that the various laws of comminution result in energy relationships which can be derived from the expression

$$\mathrm{d}W/\mathrm{d}d = -Cd^n \qquad \ldots. \; (4.10)$$

which states that the change in diameter, and hence in specific surface, brought about by a unit of energy, is a function of the particle diameter.

If the index $n$ is put equal to $-2$, and the equation integrated between the limits $d_0$ and $d_1$, the expression

$$W = C(1/d_1 - 1/d_0) \qquad \ldots. \quad (4.11)$$

is obtained; this expression being identical with equation (4.4), and so is Rittinger's law.

Kick's law may be derived by putting $n$ equal to $-1$ in equation (4.10) and integrating between the same limits as above, whereupon the expression

$$W = C \log_e d_0/d_1 \qquad \ldots. \quad (4.11a)$$

which is identical with equation (4.6a), is obtained.

In the same way, insertion of $n = -3/2$ gives

$$\mathrm{d}W/\mathrm{d}d = -Cd^{-3/2}$$

which when integrated between the same limits as previously, gives

$$W = 2C(1/d_1{}^{1/2} - 1/d_0{}^{1/2}) \qquad \ldots. \quad (4.12)$$

or

$$W = \frac{2C}{d_1{}^{1/2}} \cdot \frac{d_0{}^{1/2} - d_1{}^{1/2}}{d_0{}^{1/2}} \qquad \ldots. \quad (4.12a)$$

Bond's law thus represents a condition between those corresponding to the laws of Kick and Rittinger in relation to the energy necessary for crushing through a given reduction ratio.

It has been pointed out by Dobie(15) that if the energy $W_R$ necessary to crush a material through a reduction ratio $R$ is written in the form.

$$W_R = a + b/d_0 \qquad \ldots. \quad (4.13)$$

where $d_0$ is the initial size of the material, then the laws of Rittinger and Kick are contained therein.

Thus,

$$W_R = b/d_0 \qquad \text{(Rittinger's law)}$$

and

$$W_R = a \qquad \text{(Kick's law)}$$

furthermore

$$W_R = b/\sqrt{d_0} \qquad \text{(Bond's law)}$$

Since there is a reasonable amount of evidence that Rittinger's law applies moderately well for fine grinding and since Bond's law is deduced from the results of numerous test on real mills, it appears that, for fine grinding, it is necessary to have a law in which the work input, $W_R$, to bring about a given change in the reduction ratio, should be a function of the fineness of the material and, furthermore, $W_R$ should increase with increasing fineness. On this basis Carey and Stairmand(16) have suggested that, in order to provide a criterion for the performance of milling machinery, a crushing coefficient which increases with fineness and is related to the size distribution of the particles produced by the crushing is required. In this case the crushing coefficient is defined as the energy corresponding to a given reduction ratio, and so corresponds to $W_R$ above.

In order to obtain such a "crushing coefficient", Carey and Stairmand made use of the similarity in frequency distributions of crushed products and so developed the principle of "associated energy".

The characteristic grading produced by free crushing is termed a "natural" for the substance, and it was found that if the largest particle of a natural be crushed and added to the remainder of the distribution, the resulting grading is again a "natural". By summating the energy required to crush from one natural grading to another smaller one, it is possible, by laboratory experiments, to obtain the energy required for free crushing between natural distributions of given maxima sizes. Thus, for example, if it is assumed that a "natural" with a maximum size of 1 metre represents zero associated energy, and it is found that the energy required for successive crushings to produce a "natural" of maximum size 1 cm is 0·5 kWh/ton, and that a further addition of 0·2 kWh/ton produces a "natural" with a maximum size of 1 mm; then the energy associated with a natural of 1 cm maximum size is 0·5 kWh/ton and the energy associated with a natural of 1 mm maximum size is 0·7 kWh/ton.

In order to derive values for the associated energy, free crushing was effected by crushing between horizontal platens with a limited reduction ratio; a load versus deflection curve being used in order to compute the energy input to the crusher. By use of the figures so obtained for the associated energy, a figure for the mechanical efficiency of any grinding process may be obtained from the relationship.

Mechanical efficiency of grinding

$$= \frac{\text{Energy associated with product} - \text{Energy associated with feed}}{\text{Energy used by the mill}}$$

In the absence of established laws of comminution, the methods of Bond and of Carey and Stairmand provide a means of deriving a criterion of performance for milling machinery. The derivation of values for the associated energy or work index is tedious and the use of sieve analyses for the size frequency determinations limits the fineness of grinding to which the method can be applied, but, nevertheless, these methods appear to form useful bases for the design of crushing machinery.

Turning now to the efforts which have been made to establish the validity of the various laws of crushing by experiment, it must be stated at once that much of the work must be treated with reserve.* The reason for this caution is that the measurement of both the energy input to the crushing system and of the fineness characteristics of the product are problems of the utmost difficulty.

→ Consider first the difficulties involved in the determination of the fineness of the crushed material. The use of

* επιφυλάξεις,

sieves involves extrapolation of the grading curves into the subsieve range and, since very high specific surface is associated with the smaller sizes, small errors in the extrapolation will give rise to large errors in the calculated surface of the product. Theoretical difficulties also arise in this connection. Thus, for example, even if the external surface could be determined with absolute accuracy, the existence of internal surface within a particle of any real substance tends, to some extent, to reduce the particle to an aggregate of smaller particles. Thus it would appear that the surface which should be measured is the external surface plus the surface of any cracks which are sufficiently wide for the cohesive forces, between the two boundaries of the crack, to be effectively zero.

At first sight it would appear that a better approximation to this true surface would be obtained by the use of the adsorption method of surface measurement. Unfortunately, however, the theory of the adsorption method is still in an incomplete state and in consequence no more reliance may be placed upon the adsorption method, as an accurate measure of surface, than any other. Even if this were not the case, however, there still remains the question of whether the adsorbed molecules wedge open narrow cracks; and so whether adsorption methods assess* surface which is not strictly free surface in the present sense. Thus, it must be admitted that no known methods exist, for the determination of the size or specific surface of the smaller fractions, which are free from criticism.

*[illegible]

The method of solution of the particles, adopted by Martin(17) and by Gross and Zimmerly(18) is also open to criticism since, for example, it is known that a crystal is dissolved at different rates on faces parallel to and perpendicular to the lattice planes. Hence, a figure for the surface area, based on the average rate of solution, without a certain knowledge that the particles are geometrically similar, is unreliable.

Turning now to the measurement of the energy supplied to the crushing process. It will at once be appreciated that the energy to which reference is made in the equations of comminution is the energy usefully expended in the fraction of the particle and *not* that which is in the crushing system as a whole. Consider, for example, the measurement of the energy transferred to a single particle by a drop weight. The initial potential energy of the weight may be determined with considerable accuracy and, by the use of modern laboratory methods, the height of rebound of the hammer may also be determined. Thus, the energy given up by the weight may be determined with adequate accuracy.

The energy given up by the hammer is, however, in part converted to strain energy, and thence to heat, in part used to increase the surface energy of the powder, in part to give kinetic energy to some of the particle, in part to sound energy and in part to "pelletize*" some of the product. If, however, this heat energy could be determined, then, as a first approximation, the energy used to fracture the particle could be regarded as the difference between the energy given up by the weight and the heat energy rejected. It is easily shown, however, that the determination of this quantity of heat is extremely difficult. Thus, if it is assumed that the hammer weighs 10 lb and falls through, say, 3 ft, and also that the metal masses (hammer, anvil, etc.) which will be heated weigh 20 lb, then a simple calculation shows that, if *all* the potential energy were converted to heat, the average temperature rise would be about 0·05°F. Thus it is almost impossible to make a heat balance with worthwhile accuracy. For this reason, the analysis of the results of crushing tests are usually based on the assumption that the whole of the energy lost by the hammer is expended in fracture, but such an assumption is clearly of doubtful validity.

A further complication is that it is possible that the geometry of the system and the rate of loading of the particle have, for a given energy density, some influence on the fineness of the product obtained.

With the drop-hammer apparatus, the energy, although available in the hammer, is apparently not effectively

* pelletize =

transferred to the particle. Thus it would appear that the energy transfer process is, or can be, a controlling factor.

This view is supported to some extent by the statement by Hall,(19) that the Hadsel mill, in which ore is lifted and then dropped on to steel plates, has a power consumption of 8–12 kWh/ton which "compares favourably with ball-mill consumption for feed of $\frac{3}{4}$ in."

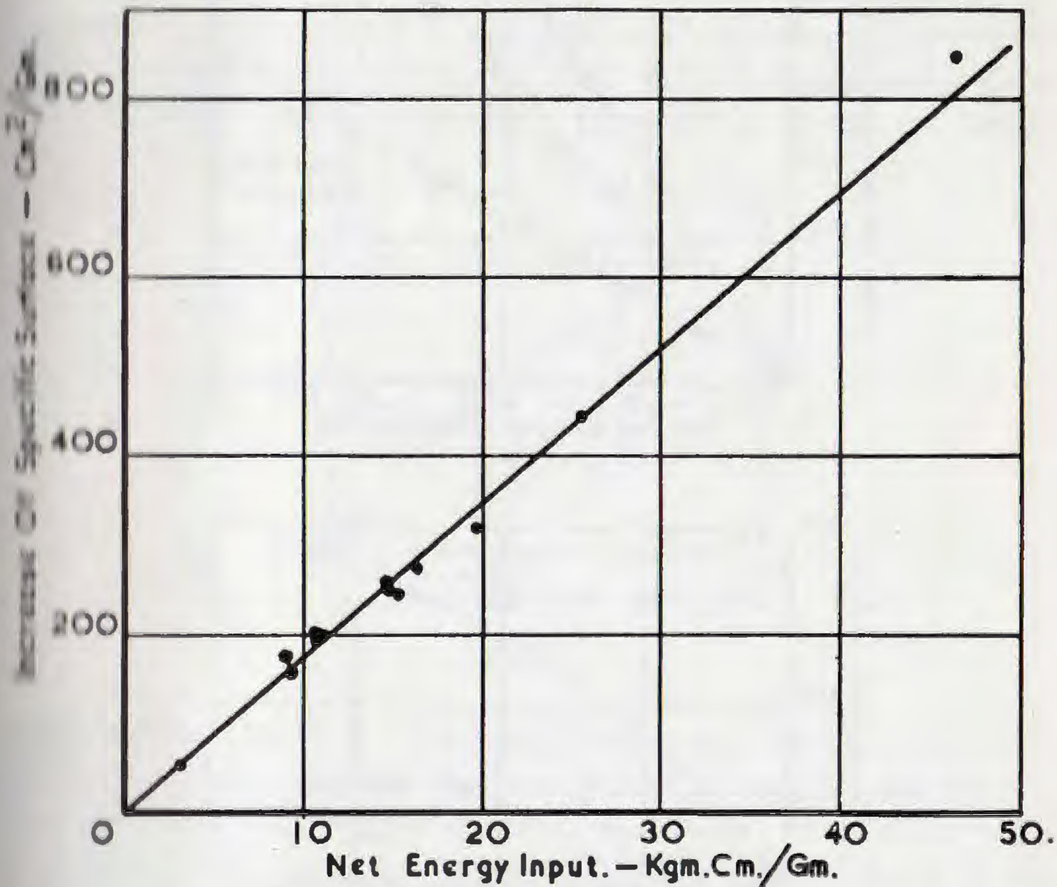


FIG. 4.5

Tests upon quartz, by use of a drop hammer, carried out by Gross and Zimmerly(18) show a linear relationship between the new surface produced and the energy input based on the energy lost by the hammer. These results are shown in Fig. 4.5.

Kwong and Piret,(20) using a drop-weight apparatus similar to that of Gross and Zimmerly, also carried out crushing tests, but in this case the air permeability method was used for the determination of the specific surface. These workers also found a constant relationship between work input and surface produced for glass, fluorite and calcite, Fig. 4.6a, but, for salt, which is a softer mineral, a

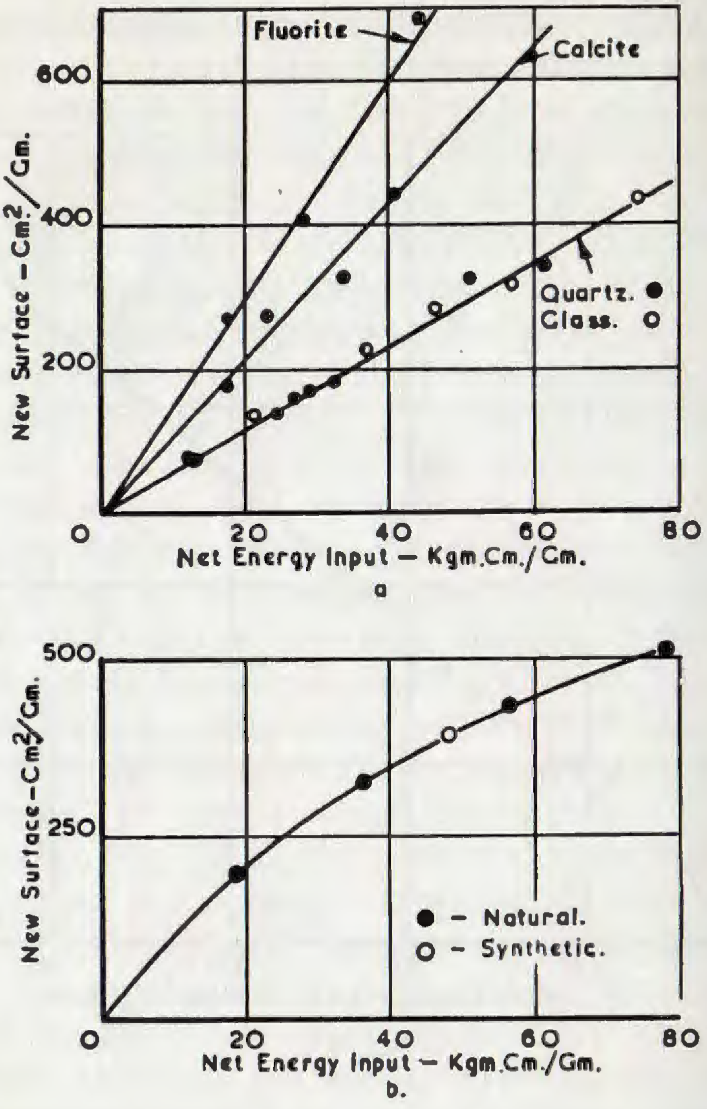


FIG. 4.6

non-linear relationship (Fig. 4.6b) was found. This result indicates a decrease in the amount of surface formed, per unit input of energy, for the larger values of specific surface. Plastic deformation of the salt crystals was suggested as the reason for this. The work of Hüttig(1) is of interest in this connection, since he suggests the presence of a

? inhomogene at coarse particles

"bond spectrum" to explain similar results obtained for the grinding of salt in a ball mill. These results are also in accordance with the work of Smekal(9) mentioned previously.

In order to carry these investigations into still finer size ranges, Kwong and Piret(21) made use of an adsorption technique for specific surface measurement, with ethane as the adsorbent. For the harder minerals, they found an increase in the energy input per unit increase in surface area for increasing fineness (Fig. 4.7) but claimed that this revealed no discrepancy with their previous work using permeability techniques, since the results were extended into much finer size ranges. This work also shows that over the size range studied, the ratio of area by adsorption to the area by permeability is fairly constant at about 1·9–2·0. Work was also carried out on the slow compression crushing of particles, and in this case, the work input was measured by computing the area under the curve obtained by plotting the displacement of the crushing surfaces against the applied load. The results (Fig. 4.8) show clearly that the ratio of new surface to energy is greater for slow compression crushing than for impact crushing. This deviation between the results was explained tentatively by changes in the usage of energy in repeated drops of the ball of the impact crusher.

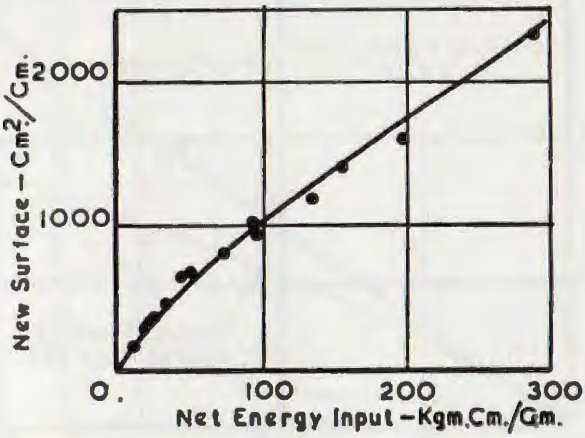


FIG. 4.7

If it is assumed that the average stress concentration in the material is a function of the energy input for each drop of the ball, and that a critical stress concentration is necessary for fracture to occur, then initially, in the drop-weight crusher, the stress concentration in the bed is high, and therefore the energy is used efficiently. However, with successive drops the effective energy input per drop decreases by 40–50% due to the frictional losses between the particles, and so the average stress concentration, and consequently the number of fractures, decreases.

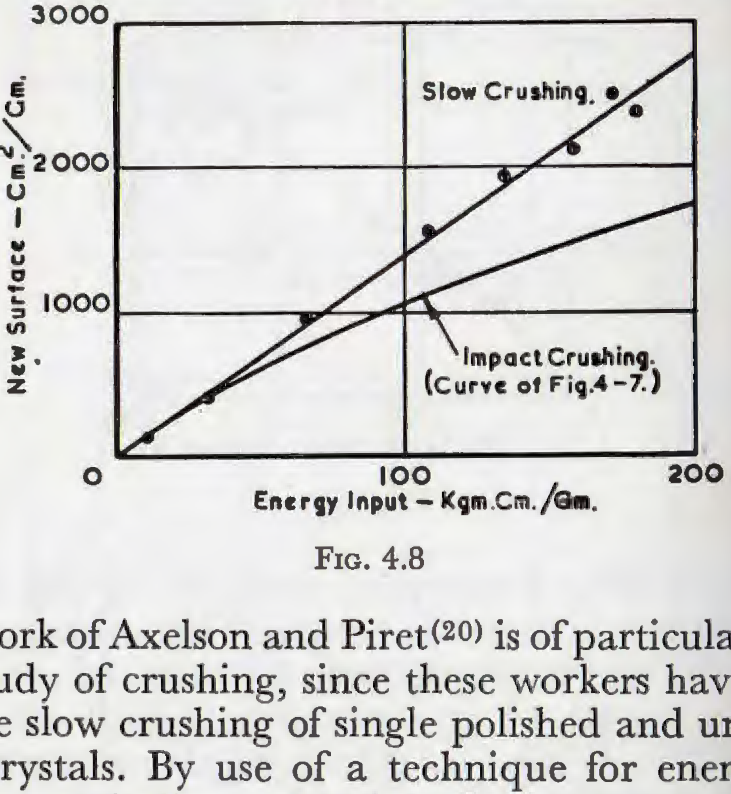


FIG. 4.8

The work of Axelson and Piret(20) is of particular interest in the study of crushing, since these workers have investigated the slow crushing of single polished and unpolished quartz crystals. By use of a technique for energy input measurement similar to that described above, it was found that the necessary stress for fracture lay between 9780 and 500 $kg/cm^2$, the natural crystals tending to have the larger stress for fracture. It was observed that although the surface created is greater the greater the energy concentration (kg cm/g), the surface created per *unit energy input* decreases for increasing energy concentration, as shown in Fig. 4.9. The hypothesis used in order to explain these results is that when fracture occurred at low energy concentrations, the fracture was caused by high local stress concentrations at relatively few points; the elastic deformation energy, which reappears mostly as heat, is thus small compared with the surface produced. When fracture occurs at high energy concentrations, however, fracture starts at a number of points, since more points are at a high level of energy concentration, and since the fracture at most points will proceed only a small distance before meeting a further line of fracture a relatively high percentage of the elastic energy will reappear as heat. The efficiency of the process is therefore small compared with the process at low energy concentrations. This indicates the importance of the internal structure on the crushing process, as previously discussed.

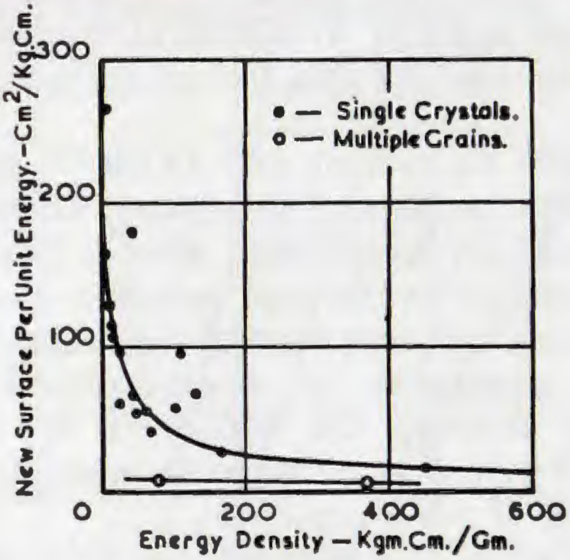


FIG. 4.9

Fairs(21) also carried out experiments on crushing using apparatus similar to that of Gross and Zimmerley.(18) He criticized the results of Kwong and Piret for impact crushing on the grounds that since, in that work, relatively deep beds of particles were used, pelleting resulted; with, consequently, departure from the free crushing conditions which ideally pertain in most types of crushing machinery. Using the permeability technique for surface measurement, linear relationships were found to exist between energy input and new surface area for Cumberland Barytes, North Wales limestone and Durham anhydrite. An adsorption technique, with krypton gas as the adsorbate, was also used to measure the total surface, but no evidence of a constant ratio between the surface by adsorption and permeability was found; in fact the ratio was found to decrease with increasing fineness.

The question of pelleting of the product, mentioned earlier, is of some interest since the researches of Bowden and Tabor⁽²²⁾ show that impacted surfaces may become strongly welded together. It is evident that if this happens there will be a considerable loss of surface; with consequent increase in particle size.

Researches by Mackenzie and Milne⁽²³⁾ show that the X-ray diffraction pattern of muscovite becomes more diffuse as grinding proceeds until, after 8 hours, no sharp pattern remains. After further grinding the pattern reappears but has a different form to the original. This result suggests the possibility of re-crystallization of these materials and, although the evidence is not conclusive, it is suggestive that the same phenomenon might occur in the fine milling of other materials.

In the work which has been discussed, in this chapter, it has been assumed that there is a similarity in the crushing pattern of materials comminuted by slow crushing and by blows of various magnitudes. However, none of the workers appears to have attempted to establish experimentally the truth of the assumption. This raises the following question: If a given input of energy to a particle produces a given increase in surface and assuming, for example, that an input of twice the energy produces twice the surface increase, then are the size frequency curves for the two products similar, with the scale of particle size changed, or is there no relationship between the products, other than a two-to-one ratio of the increase in fineness?

This question has been studied by Andreasen,⁽²⁴⁾ however, who has shown that the cumulative curves of the products obtained by crushing geometrically similar bodies are orthogonally affine,* and hence it follows that:

(1) The size distributions for the materials are geometrically similar.
(2) The specific surfaces of a series of products vary inversely as the abscissae of the cumulative curve, for any specified value of the ordinate.

A series of product curves for various porcelain cubes, as determined by Andreason, are given in Fig. 4.10, and it

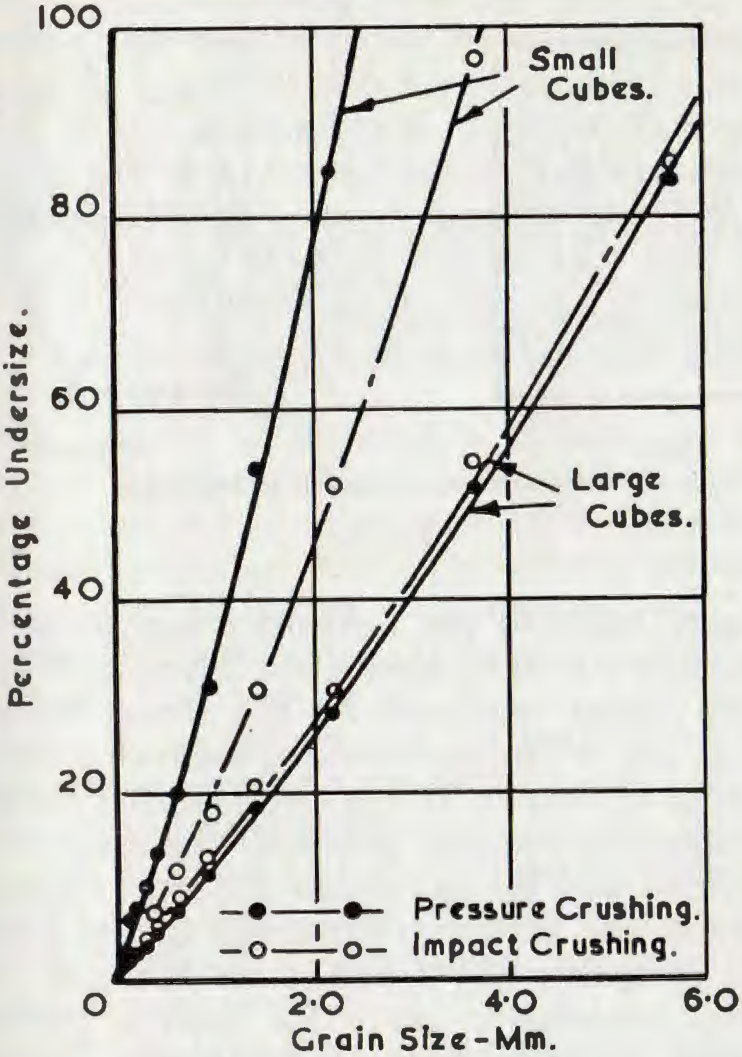


FIG. 4.10

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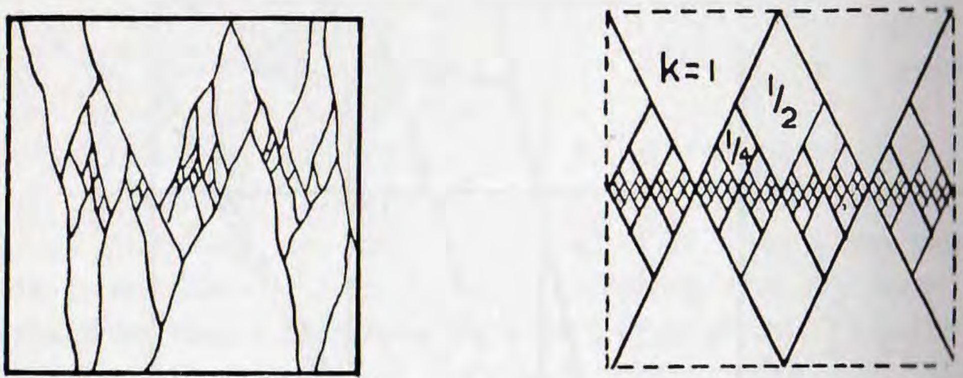
* For the present purpose it is only necessary to interpret "orthogonally affine" as meaning that the curves can be brought into coincidence by a suitable change of the independent variable (size) scale.

is seen at once that, for all practical purposes, these are orthogonally affine. These curves are also representative of curves obtained by tests on feldspar cubes, glass cylinders, etc., and so it may be stated that there is some experimental proof that geometrical similarity holds in the fracture pattern. The present authors, Rose and Sullivan, have, in the experiments on the crushing of glass spheres, also found that the product curves are, roughly, orthogonally affine and, in this respect, confirm the work of Andreasen.



Real. Idealized.

Fracture Patterns According to Andreasen.

FIG. 4.11

Andreasen has also put forward a logical explanation for the existence of such similarity. Thus, in Fig. 4.11a is shown the pattern obtained by the actual fracture of a cube and in Fig. 4.11b, an idealized fracture pattern which can be deduced from it. If it is assumed that the principal plane of failure is parallel to one of the diagonal planes, then it follows that the grain size of unity comprises 50 % by volume of the material, the grain size of $\frac{1}{2}$ comprises 25 %, of $\frac{1}{4}$ comprises $12\frac{1}{2}$ % and so on. It is also clear that the weight cumulative curve for such a product is a straight line passing through the origin. Thus, there appears to be a fundamental reason for the type of curve obtained by Andreasen. His work is also interesting in that the treatment leads, at once, to Kick's law, and Andreasen explains that the closer approximation to Rittinger's law which is obtained during fine grinding arises from a loss of efficiency of the mill when grinding finer particles.

An interesting point which emerges from the analysis of Andreasen's drop-weight tests is that, for the larger cubes, the agreement between the product size obtained, for same energy input, by slow crushing in a press and by drop weight, is very close. For the smaller cubes, however, the product size from slow crushing is in very close agreement with that calculated from the slow crushing of the large cubes, and hence with that obtained by impact crushing of the large cubes, but the product obtained by the impact crushing of the small cubes is far too coarse, although the curves are still orthogonally affine. Since Andreasen used a fixed height of drop for the hammer, and hence a fixed velocity at impact, this result suggests that the crushing process is velocity dependent and that for true similarity in crushing it is necessary for the size and other characteristics of the particle to be in a definite relation to the velocity of the hammer.

The fair success of researches of Martin,(25) Rosin, Rammler and Sperling,(26) and Müller(27) and others, into the possible existence of a general law of size distribution for powdered materials also suggests the existence of a fundamental mechanism of comminution, but "laws" of general application do not, at the present time, exist.

The complicated process of fracture and the lack of homogeneity in the structure of the majority of materials to be ground, particularly minerals, makes the development of a general relationship between size distribution, size reduction and energy input extremely difficult. Thus the process of deducing energy relationships is largely empirical and related to particular conditions and types of reduction machinery.

The present position regarding the investigation of the laws of comminution can probably be fairly summed up as follows. The problem involves extreme difficulty in the fundamental measurements. In addition, apart from any questions of the accuracy of the actual measurements, much of the existing work must be treated with reserve since it is generally impossible to differentiate between energy put in to the grinding system and that usefully expended in comminution. Many of the published results are further confused by the crushing of deep beds of particles; there being no theoretically sound reason why the crushing of the particles in a bed should obey the same rules as the crushing of a single particle.

In general, questions arising from imperfect nature of individual crystals and the inhomogeneous nature of many industrially important materials have not received the attention which the subject merits. The truth of the assumption of geometrical similarity between a particle and its progeny has not been adequately investigated. The laws of Rittinger and of Kick are possibly extremes, between which most practical ideal crushing processes lie. It is scarcely possible at present, however, to state the conditions for one or other of these laws to be applicable; nor is it possible to state, in any particular case, the law to which the actual grinding operation will approximate most closely. It appears fairly safe to say, however, that coarse grinding will approximate more closely to Kick's law and fine grinding more nearly to Rittinger's law.

The extent to which the design of a mill, and the physical dimensions of the crushing elements influence the crushing characteristics of a given material is uncertain. It appears that, in general, the efficiency of transfer of energy to the particle falls with decreasing particle size, and this factor alone makes the overall behaviour of most types of mill approximate to the "Rittinger" type of law; even though the fundamental crushing process might be more nearly in accordance with a law of the "Kick" type.

Finally, it may be said that the subject of comminution offers a wide field for useful, interesting and fundamental investigation.

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CHAPTER 5

# THE PROCESS OF GRINDING WITHIN A MILL

In the previous chapter, the various "laws" of comminution which have been propounded have been studied in relation to the physical process of size reduction and to the available results of experiments into the fracture of homogeneous and heterogeneous materials by impact and by slow compression. In this study it is seen that no satisfactory "law" of comminution has yet been deduced but that a number of more or less empirical expressions are available for the guidance of the designer of crushing machinery.

The present chapter will be devoted to a study of the process of grinding within a real mill. Since within a "tumbling" type of mill the size reduction can result from direct crushing, from shearing or abrasion between the particles in the feed, between the particles in the feed and the mill bodies or between the particles and the mill lining, the process is much more complicated than in the case of simple "free" crushing conditions considered previously. In addition to this complication, however, the motion of the ball charge itself is extremely complex. Thus the precise mode of transfer of the energy to the crushing surfaces cannot be simply represented.

For these reasons, a rigid theoretical analysis of the parameters involved in the milling process becomes impracticable, and recourse is frequently made to pilot scale experiments in order to determine optimum milling conditions. Nevertheless, considerable experimental and theoretical work has been done on the crushing process in the "tumbling" mill, and it is desirable that these data should be analysed and an effort made to establish the extent to which the performance of a mill may be predicted from theoretical considerations. The experimental study of the performance of mills has, in general, been along the following lines:

(1) The energy relationships and theoretical efficiency of the ball mill from surface energy and thermodynamic considerations.
(2) The size distribution of the product in relation to the mill parameters.
(3) The rate of crushing in relation to the mill parameters.

Actually these three divisions are not independent since, clearly, if there is a definite relationship between the energy input to a particle and the change of surface area, or reduction ratio, it follows that a given mill, to which the energy input is constant, will lead to a definite rate of crushing; the rate being a function of the relevant mill parameters. In fact the increase in the surface area of the material, produced by the input of a unit of energy to the mill, is increasingly accepted as a criterion of performance for fine milling operations. Without doubt this is due to the technological importance of specific surface in the sub-sieve range, and to the establishment of reasonably satisfactory techniques for specific surface measurements. It is for this reason that Rittinger's law, even though lacking a sound theoretical basis, is widely accepted as a basis for the calculation of the performance of such machines.

Let us apply these ideas to the derivation of an equation for the rate of grinding in a mill. It follows that if, as appears reasonable, the change of specific surface $S$ cm$^2$/cm$^3$, within a mill depends upon the diameter of the mill, upon the ball diameter and upon the mean diameter of the particle, $D$, $d$ and $b$ respectively, upon the density of the balls and the pulp, $\varrho$ and $\sigma$ respectively, on gravity, $g$, on the speed of the mill, $N$, upon $u$ the diameter of the exit from the mill, upon $E$, the energy necessary to change the

$\rho$ = density of grinding media

surface of the material by unit amount and upon the parameters $J$, $U$, $V$ and $Z$, where $J$ is the fraction of the mill volume occupied by the balls (or rods) and the space between them, $V$ the volume occupied by the powder (or pulp) charge, including voids expressed as a fraction of the volume between the balls (or rods), $U$ is the ratio of solid volume to total volume of the pulp, and $Z$ is the ratio of circulating load to the total load when closed circuit operation is considered, and clearly, $S$ will depend on the time of milling, $t$.

Then we may write symbolically:

$$S = \phi(D, d, b, \varrho, \sigma, g, N, E, t, J, u, U, V, Z) \quad \ldots . \quad (5.1)$$

where, as before, $\phi$ denotes "a function of".

The application of the principles of dimensional analysis, already illustrated in Chapter 3, leads to the following equation.

$$(S.D) = \phi\left\{\left(\frac{D^2 g\varrho}{E}\right), \left(\frac{d}{D}\right), \left(\frac{b}{D}\right), \left(\frac{\sigma}{\varrho}\right), \left(\frac{u}{D}\right), \left(\frac{g}{DN^2}\right), \left(\frac{gt^2}{D}\right), (J), (U), (V), (Z)\right\}$$

which, as before, may be written

$$(S.D) = \phi_1\left(\frac{D^2 g\varrho}{E}\right) \cdot \phi_2\left(\frac{d}{D}\right) \cdot \phi_3\left(\frac{b}{D}\right) \cdot \phi_4\left(\frac{g}{DN^2}\right) \cdot \phi_5\left(\frac{gt^2}{D}\right) \times \phi_6\left(\frac{\sigma}{\varrho}\right) \cdot \phi_7(J) \cdot \phi_8(U) \cdot \phi_9(V) \cdot \phi_{10}(Z) \cdot \phi_{11}\left(\frac{u}{D}\right) \quad \ldots . \quad (5.2)$$

This equation (5.2), is the dimensionless equation giving the rate of production of specific surface in a tumbling mill.

Purely theoretical considerations are insufficient to take the analysis further, but, in practice, the various dimensionless relationships may be established by reference to experimental data.

Before proceeding with such an analysis of experimental data, however, a digression will be made to introduce briefly a theoretical treatment of the internal dynamics of the ball mill propounded by one of the present authors, Rose.(1) This, as far as the writers know, is the only attempt to deduce the general behaviour, including such factors as ball wear, from purely theoretical considerations. Furthermore, this theory can best be studied in conjunction with the analysis of the experimental results which is necessary for the evaluation of the above dimensionless equation. This theoretical treatment is of necessity extremely tentative since the dynamics of a mill are of such complexity as to defy rigid analysis. Nevertheless, it is believed to form the basis of a more precise treatment.

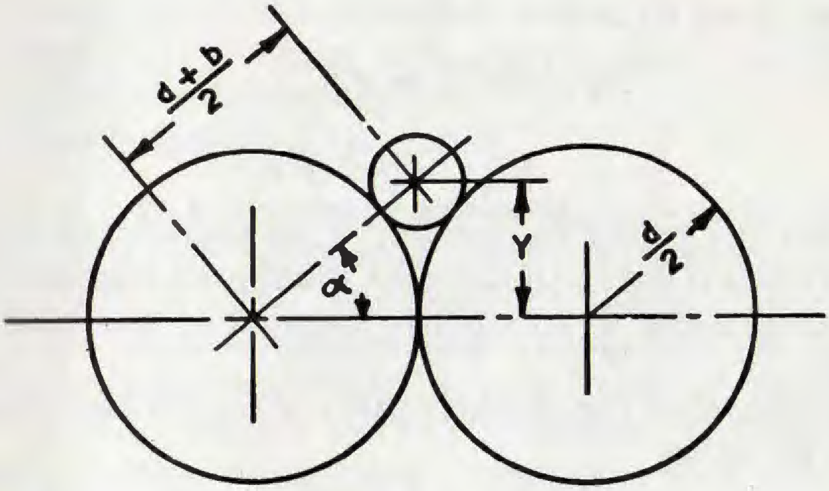

FIG. 5.1

The basic idea underlying this treatment is that the whole behaviour of a ball mill depends on the probability of a particle arriving into a "zone of operation" between two balls, and on the probability of occurrence of some process when it arrives there. Outside this zone, no operation, such as crushing or metal wear, which demands the trapping of a particle between two balls can take place.

From Fig. 5.1 it follows that the zone of operation is a circular area of radius $Y$, where $Y$ is given by

$$Y^2 + \left(\frac{d}{2}\right)^2 = \left(\frac{d+b}{2}\right)^2 = \frac{d^2}{4} + \frac{db}{2} + \frac{b^2}{4}$$

or

$$Y = \sqrt{(db/2)} \qquad \ldots . \ (5.3)$$

provided $b$ is small compared with $d$; $d$ and $b$ being the diameter of the ball and of the particle respectively.

Now the maximum number of particles, $N_{max}$, which can be in the operating zone are those which just form a ring around the contact point.*

Thus, from equation (5.3),

$$2\pi\sqrt{(db/2)} = dN_{\max}$$

or

$$N_{\max} = 4{\cdot}5(d/b)^{1/2} \qquad \ldots . \ (5.4)$$

In fact the probable number $N_p$ of particles in the zone of operation is less than this; the maximum number occurring when there is an infinite number of particles in the system.

It is suggested that a likely form of the probability function is given by

$$\phi(N) = 1 - e^{-kV} \qquad \ldots . \ (5.5)$$

and if it is assumed that about 40% of the maximum number of particles are present when the void space between the balls is just filled, that is when $V = 1$, then

$$N_p = 4{\cdot}5(d/b)^{1/2}(1 - e^{-0{\cdot}5V}) \qquad \ldots . \ (5.6)$$

Thus, this equation gives the probable number of particles in the "zone of operation", in terms of the ball diameter and the quantity of powder present in the mill.

Consideration is then given to the equilibrium number of particles, $N_e$, embedded in the surface of a ball and this is clearly given by

$$dN/dt = N_i - N_o \qquad \ldots . \ (5.7)$$

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* The number would scarcely be expected to be proportional to the area of the circle of radius $Y$, since a particle inside the circle automatically relieves some of the others of load, but all of the particles in a ring could be loaded.

* to tear out = pull violently out = τραβώ

where $N_t$ is the number pressed in to the metal surface per unit time and $N_o$ is the number of particles torn* out per unit time.

Certain plausible probability functions are derived for these quantities and so an expression for the equilibrium number of particles embedded in the surface of a ball is obtained; this expression including terms to cover the hardness of the ball relative to the hardness of the particle.

It may then be stated that a relatively very soft particle will be crushed when trapped between

(a) Two bare metal surfaces,
(b) One bare surface and one surface "armoured" with embedded particles,
(c) Two "armoured" surfaces,

whereas a relatively very hard particle will be crushed only between two "armoured" surfaces (it will be pressed into an "unarmoured" one) and probability functions to cover these possibilities are introduced. On the basis of these functions, expressions for the rates of milling, in terms of the various mill parameters, are deduced. Similarly it may be deduced that metal wear will occur when, during relative sliding motion between a pair of balls, the "armoured" portion of the surface of one ball traverses the "unarmoured" portion of the surface of the neighbouring ball. From a knowledge of the equilibrium number of particles embedded in the surface of a ball the probability of such "armoured" to "unarmoured" contact is computed and expressions for metal wear deduced.

The analysis leads to extremely complex expressions for the rate of crushing; expressions which do not lend themselves to a general form of solution. For the two boundary conditions, in which the particles are either very hard or very soft compared with the balls, however, the equations, under certain assumptions, become tractable and the solution for these two conditions can be written in the form

$$S^a = Kt \qquad \dots\dots (5.8)$$

where $S$ is the specific surface of the particles, $K$ is a constant, and $a$ is $\frac{1}{2}$ for very soft materials and 4·5 for very hard materials; all other cases falling between these two conditions.

It is interesting to note that the case in which $a=\frac{1}{2}$ corresponds to the "Bond Law of Comminution" discussed previously. Furthermore, for some intermediate hardness there is an *approximately* (but not strictly) linear relationship between the increase in specific surface and the milling time.

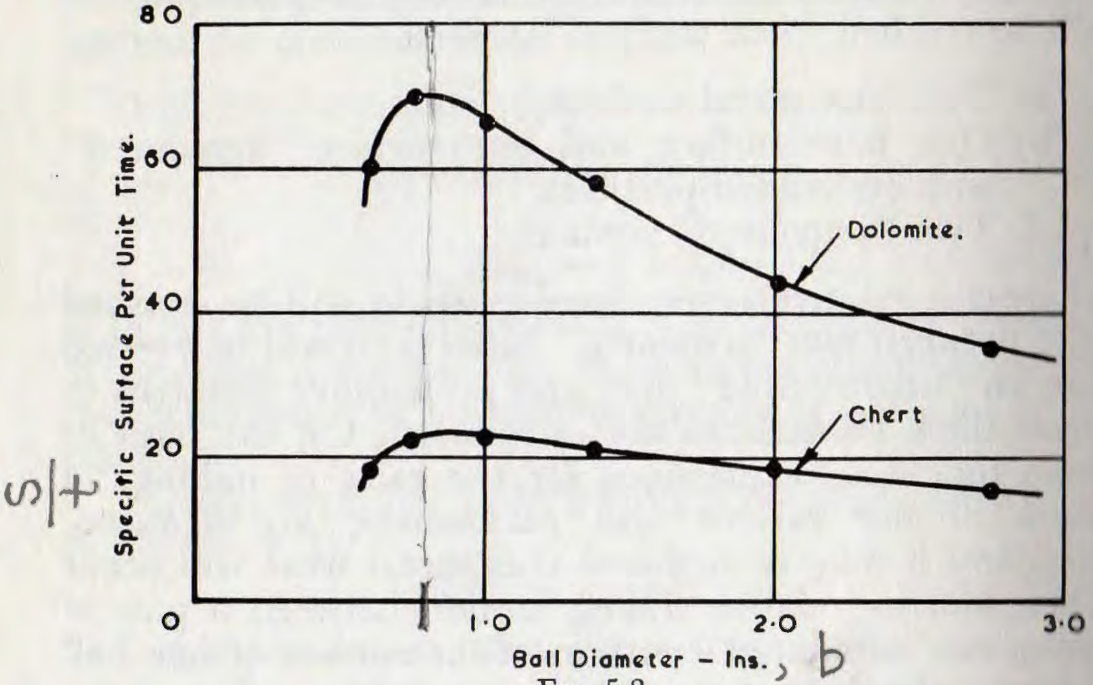


FIG. 5.2

The analysis also reveals that the rate of grinding increases with decreasing ball size; the improvement brought about by a given reduction of size being greater with soft material. This result is clearly confirmed by Fig. 5.2, in which results of tests by Coghill and Devaney(2) on chert and dolomite, these materials having hardnesses in the ratio of about 3 to 1, are plotted.

Analysis of the fineness of grind in relation to the powder charge reveals that within limits, the fineness of grinding can be increased by reduction of the amount of powder in the mill; all other variables being held constant; a maximum being reached when the powder about fills one-tenth of the space between the balls. When a soft material is being ground, however, this effect is much less marked and also the maximum is reached when the powder occupies about one-half of the space between the balls. The analysis also shows that the wear on the surfaces of the balls and mill is a function of the quantity of powder present; this function reaching a maximum when about one-tenth of the space between the balls is filled and thereafter decreasing rapidly.

One further matter of interest arises in connection with the concept of a "zone of operation" in the ball mill. It follows from equation (5.3), that the radius of the zone varies as the square root of the particle diameter. Other things being equal, therefore, there is a definitely reduced probability of the smaller particle entering the zone of operation and, consequently, a preferential grinding of the larger particles would be expected; provided, of course, that the particles are neither so big in relation to the ball size that they are expelled from the wedge-shaped space between the balls nor that the balls contain insufficient energy for the fracture of the particles. In fact, the "specific grinding characteristics" deduced by Theimer and Moser(3) show a marked tendency in agreement with these conclusions. This preferential grinding would also suggest that the size distribution of the material after grinding would not be geometrically similar to that initially, but that a "tail" initially on the right-hand side of the curve would be reduced; this geometrical similarity between mill feed and product would not be expected.

From this theoretical treatment the following tentative conclusions emerge.

(1) The processes occurring in a ball mill are controlled by the probability of particles entering a "zone of operation" between two balls and by the probability of occurrence of some process after they have entered the zone.

(2) The rate of grinding of the powder is a function of the quantity of powder in the space between the balls and

this function reaches a maximum value when this space is approximately full.

(3) The fineness of grinding attained by a hard material in a given time of grinding increases rapidly as the quantity of powder between the balls is decreased; a maximum being reached when the powder fills approximately one-tenth of the space between the balls. When a soft material is being ground, however, this effect is much less marked and also the maximum is reached when the powder occupies about one-half of the space between the balls.

(4) The law giving the change of surface with time of grinding is of very complicated form and includes terms depending upon the hardness of the material undergoing grinding.

(5) The reduced probability of the smaller particles entering the grinding area would suggest that there is a preferential grinding of the larger particles. Thus it would follow that the size distribution curve of the material after grinding would not be geometrically similar to that initially, but that any "tail" initially to the right-hand end of the curve would be reduced; the negative slope of the curve thus being increased.

(6) The wear of the ball surfaces is a function of the equilibrium number of particles pressed into the surfaces and this number depends upon the relative hardness of the balls and of the particle. Thus it follows from the treatment, that there exists the possibility that *soft* balls will wear less than hard ones; since soft balls might acquire an artificial "hard" surface by reason of the hard particles embedded in the surface of the metal. This is in accordance with the practical observations of Engels,(4) who states that the bodies of mills for grinding soft material should be hard whereas the bodies of mills for grinding hard materials should be soft.

(7) The wear on the surfaces of the balls and mill is a function of the quantity of powder present, this function reaching a maximum when approximately one-tenth of

the space between the balls is filled, and thereafter decreasing rapidly.

The general validity of these conclusions will be discussed later in this chapter in connection with the study of the results of grinding tests.

Reverting now to the study of the grinding tests by use of the dimensionless equation (5.2), it is found that Gross⁽⁵⁾ has given figures for the values of "Rittinger's Constant" for galena, sphalerite, pyrite and calcite, as determined by use of a laboratory ball mill, run under specified conditions, and also figures for the Rittinger's constant as determined by the use of a drop-weight apparatus.

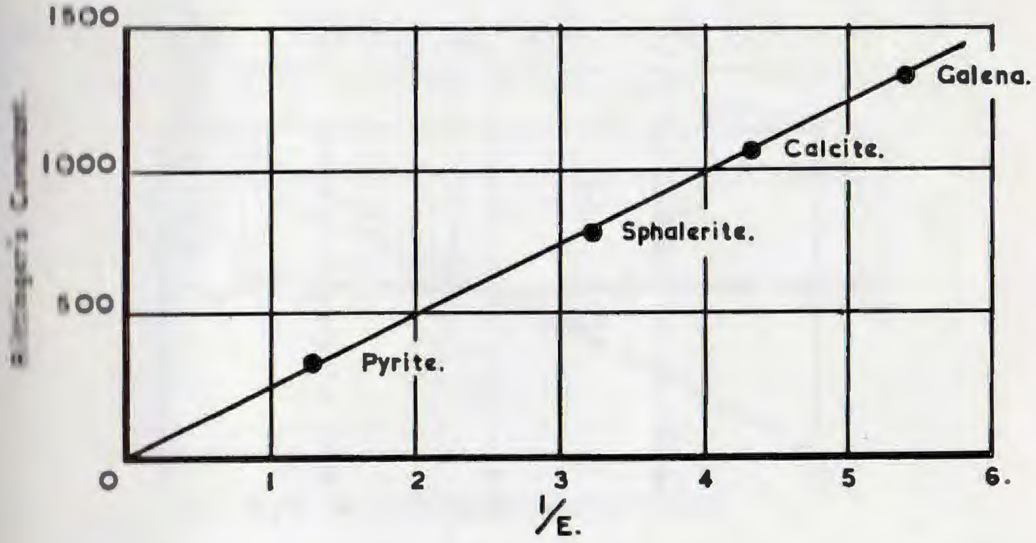


FIG. 5.3

Clearly, since, as follows from the treatment of Chapter 2, the power to drive the mill is sensibly constant, the value of Rittinger's constant for the mill is proportional to the increase of specific surfaces, $S$. Furthermore, the drop-weight figures for energy per unit increase in specific surface is directly proportional to $E$, so when the Rittinger's constant for the mill is plotted against the results of the drop weight test a curve of $S$ plotted against $1/E$ for the mill is obtained.

This is done in Fig. 5.3 and a sensibly straight line is obtained; thus it follows that the functional relationship

$$(S.D) = \phi_1(D^2g\varrho/E)$$

is, in fact, a direct proportionality or

$$(S.D) = K(D^2g\varrho/E) \qquad \ldots. \quad (5.9)$$

This relationship is for a batch mill grinding a single material.

This expression can, however, be written as

$$(S.D) = K(D^4g\varrho.1/D^2E) \qquad \ldots. \quad (5.10)$$

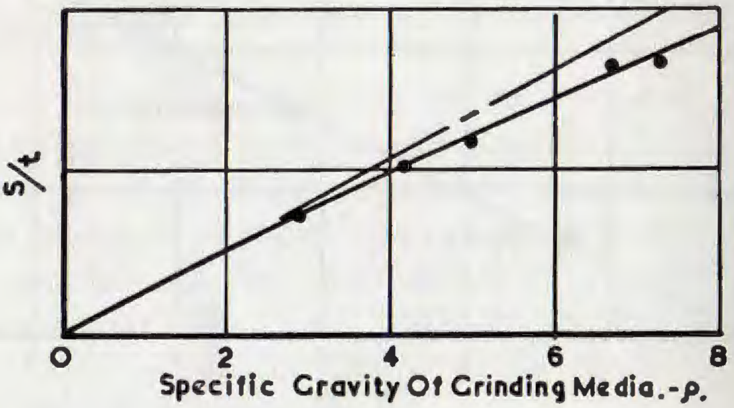


FIG. 5.4

where $D^4g\varrho$ is the potential energy in the rods and $D^2E$ is the energy associated with the powder, and a further check made on this relationship since, in a given mill, the increase in surface, $S$, per unit time will be proportional to the density of the grinding bodies, $\varrho$.

Coghill and Devaney,⁽⁶⁾ have given the results (Table 4) for the grinding of chert in a rod mill, in which the density of the rods is varied and these results are plotted in Fig. 5.4. It is at once seen that the two quantities are almost exactly proportional, as required by equation (5.10). If the deviation from linearity is assumed to arise by reason of the existence of the group $\phi_6(\varrho/\sigma)$, then this relationship may be computed. The relationship $\phi_6(\varrho/\sigma)$ so determined is plotted in Fig. 5.5, and it is seen that the influence of this group is not large and it appears probable that the difference arises from the reduced efficiency of crushing at high energy densities, which has been discussed in the previous chapter. These relationships are still further confirmed, however, by the results of comparative tests, carried out by Coghill and Devaney, in which chert and dolomite are ground by use of balls and pebbles (Table 21). These results are summarized in Table 5.1, and it is seen that the agreement is good.

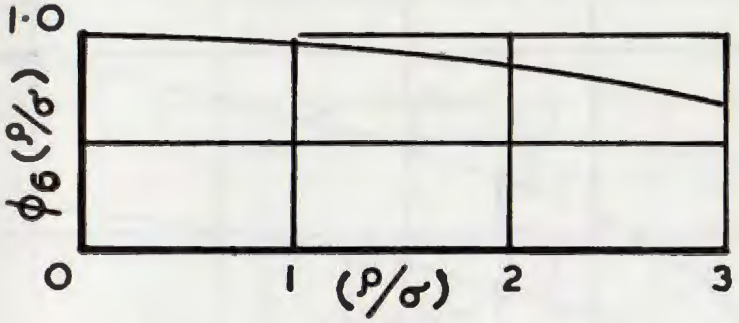


FIG. 5.5

TABLE 5.1

| Material | Density (g/cm$^3$) | | Increase in surface per hour | | Ratio of surface increase hour | Ratio for $\varrho = 7{\cdot}8$ and $\varrho = 2{\cdot}7$ From Fig. 5.4 |
|---|---|---|---|---|---|---|
| | Balls | Pebbles | Balls | Pebbles | | |
| Chert | 7·8 | 2·7 | 15·4 | 5·5 | 2·8 | 2·58 |
| Dolomite | 7·8 | 2·7 | 46·0 | 16·9 | 2·7 | 2·58 |

The function given in Fig. 5.5 is probably not highly accurate, since it has been derived from a combination of results for tests on drop-weight crushing, on a ball mill and on a rod mill, but it is of value in that it indicates the trends which may be expected to exist.

This treatment leads to the interesting idea that, for a given material to be ground there possibly exists an optimum density of material for the grinding material.

? Thus, from Chapter 3, the power $P$ to rotate a ball mill is given by

?

$$P \overset{\propto}{=} \varrho(1+0{\cdot}4\sigma/\varrho)$$
$$\overset{\propto}{=} \varrho+0{\cdot}4\sigma = K(\varrho+0{\cdot}4\sigma) \quad (5.11)$$

Also, from Fig. 5.4

$$S/t \propto 50\varrho-\varrho^2 \quad \dots\dots \quad (5.12)$$

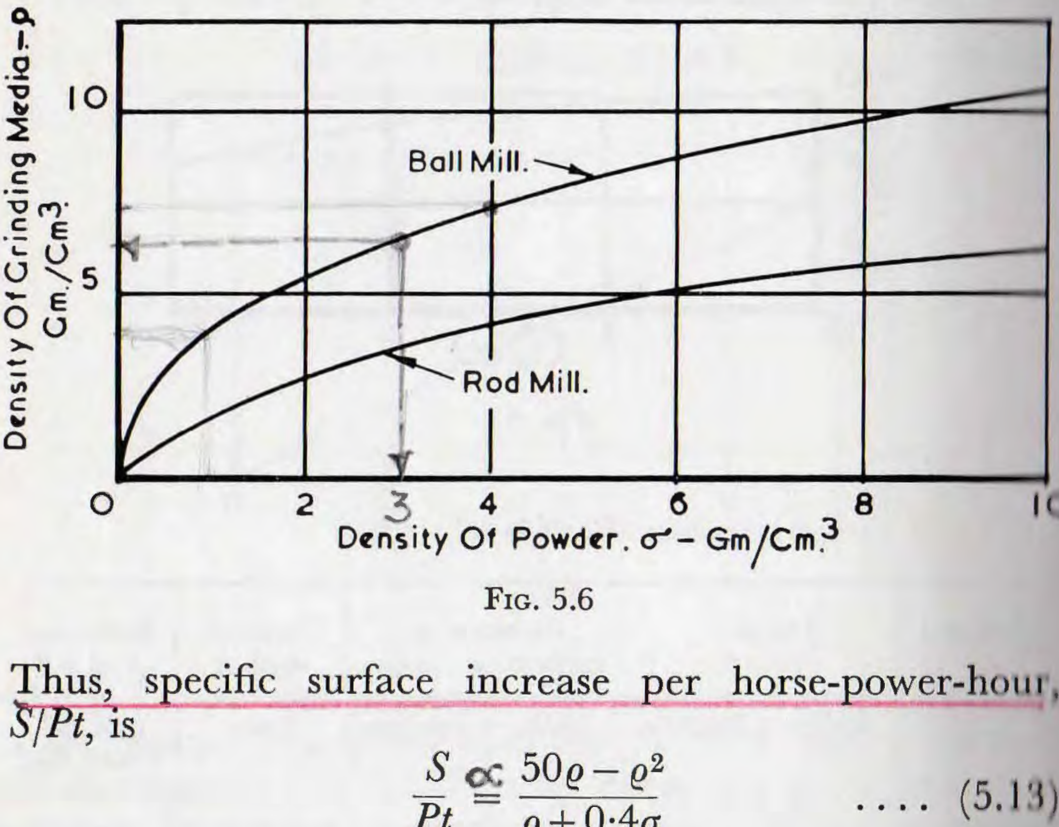


FIG. 5.6

Thus, specific surface increase per horse-power-hour, $S/Pt$, is

?

$$\frac{S}{Pt} \overset{\propto}{=} \frac{50\varrho-\varrho^2}{\varrho+0{\cdot}4\sigma} \quad \dots\dots \quad (5.13)$$

On differentiating with respect to $\varrho$, equating to zero and simplifying,

$$\varrho^2+0{\cdot}8\sigma\varrho-20\sigma = 0$$

or

$$\varrho = -0{\cdot}4\sigma \pm \surd(0{\cdot}016\sigma^2+20\sigma) \quad \dots\dots \quad (5.14)$$

From this equation the optimum values of density of balls, for grinding materials of various densities, are calculated and these figures are plotted in Fig. 5.6.

$\varrho$ = density of grinding media

In the same figure is also shown the corresponding curve for a rod mill; and it is seen that the same general form of relationship holds.

These curves are not of great practical importance since the types of materials suitable for grinding bodies are very limited, and so the range of densities available is restricted.

The curves do show, however, that the use of lumps of ore as grinding bodies is a practice which, at least on grounds of milling efficiency, cannot be recommended.

The next pair of groups which may well be studied are

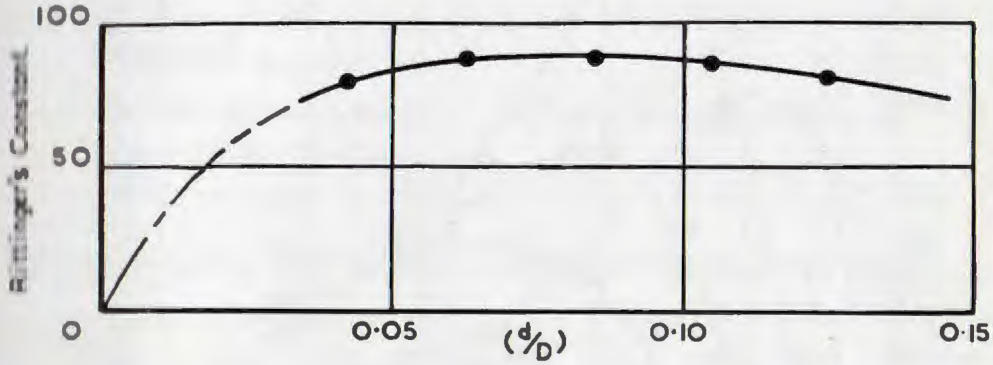


FIG. 5.7

$\phi_2(d/D)$ and $\phi_3(b/D)$ and these are most conveniently considered together. From Table 32 of the work of Gross the value of Rittinger's constant for various diameters of ball, in a mill of specified size, is given. Since the diameters of mill and balls are known a curve of Rittinger's constant against $(d/D)$, Fig. 5.7, may be plotted; this curve being equivalent to a curve of $\phi_2(d/D)$ plotted against $(d/D)$. The variation shown, however, arises in part from the variation of $(b/d)$ which occurs with the change of ball size; this variation arising from the use of the same feed material for each ball size.

Coghill and Devaney, however, give sufficient data, in Tables 27 and 28 of their work, for these effects to be separated, and the basic data from these tables are plotted

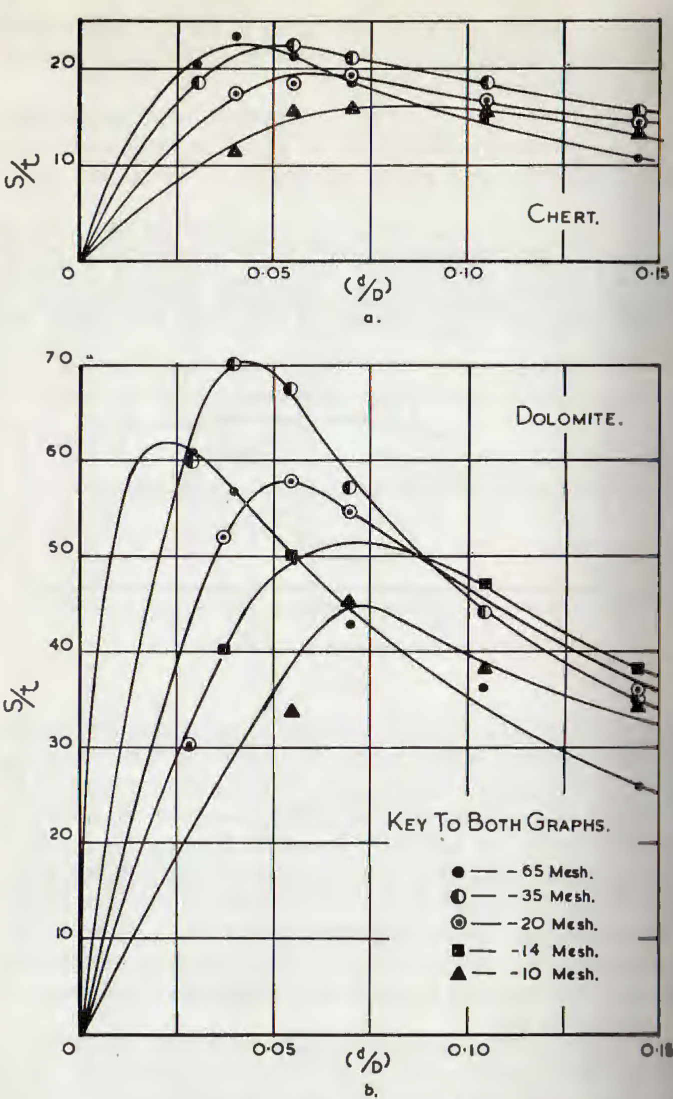

S/t
20
10
0
0·05
0·10
0·15
(d/D)
CHERT.
a.
70
60
50
40
30
20
10
S/t
DOLOMITE.
KEY TO BOTH GRAPHS.
● — -65 Mesh.
◐ — -35 Mesh.
◎ — -20 Mesh.
■ — -14 Mesh.
▲ — -10 Mesh.
0
0·05
0·10
0·15
(d/D)
b.

FIG. 5.8

in Fig. 5.8; a base of $d/D$ being used since, again, the ball diameter and the mill diameter are known. Since the time of grinding is comparatively short, it is assumed that the effective particle diameter is the average of the mean particle size of the feed and of the product; the mean particle size of the feed and product being assumed to be the sieve size which passes (or retains) 50 % of the sample. On this basis the data of Table 5.2 may be calculated.

If now the curves of Fig. 5.8 are studied, it will be noticed that the curves are similar in form and that they may be brought into coincidence by:

(1) Multiplying the abscissae of each curve by a suitable constant, such that the maxima are brought on to the same vertical line; this constant being related to the mean particle size of the material to which the curve relates.
(2) Multiplying the ordinates by a suitable factor.

Examination will show that the curves are brought into horizontal coincidence by multiplying the value of $(d/D)$ by $(D/b)^{1/2}$ and this product, after squaring, becomes $(d^2/Db)$ which may be treated as the new independent variable. This may be regarded as a correction to the $(d/D)$ term to allow for the effect of the ratio $(b/D)$ upon the effect of this term.

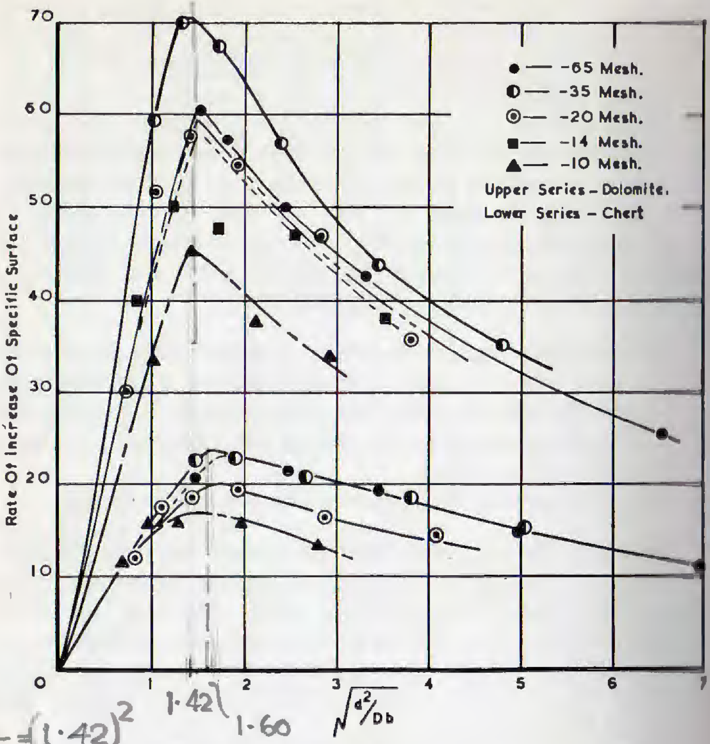
When the data are plotted for the dolomite and chert in this way, as in Fig. 5.9, good horizontal coincidence is obtained. (The root scale of this diagram is used merely to enlarge the scale for low values.) It will also be noticed that the maxima for these materials occur at, as closely as can be expected, $d^2/Db = 2{\cdot}00$ $(= 1{\cdot}42^2)$ for the dolomite and $2{\cdot}56$ $(= 1{\cdot}60^2)$ for the chert.

Since the mill is 19 in. diameter, these values give the equations:

$$\left.\begin{aligned} d^2 &= 38{\cdot}0b \quad \text{for dolomite} \\ \text{and} \qquad d^2 &= 48{\cdot}5b \quad \text{for chert} \end{aligned}\right\} \quad \ldots . \quad (5.15)$$

$d = \sqrt{38} \cdot \sqrt{b}$

$d = \sqrt{48.5}\sqrt{b}$



$\frac{d^2}{Db} = (1{\cdot}42)^2$ 1·42 1·60

FIG. 5.9

for the optimum size of balls, in terms of particle size.

This should be compared with the expressions

$d \propto \sqrt{b}$

$$\left.\begin{array}{ll} d^2 = 35b & \text{for dolomite} \\ d^2 = 55b & \text{for chert} \end{array}\right\} \quad \dots\dots (5.16)$$

given by Coghill and Devaney,(6) and it is seen that the agreement is satisfactory. These formulae are also similar to those developed by Starke.(7)

If now the ordinates of the curves of Fig. 5.9, for any specified value of $(d^2/Db)$, are plotted against the corresponding values of $(b/D)$, the curves of Fig. 5.10 are obtained.

Again it will be seen that the curves may be brought into almost perfect coincidence by multiplying the ordinate for chert by 2·8 and, as a confirmation of the accuracy of the treatment, this is almost exactly the ratio of the rates of milling obtained from the independent tests, making use of balls and pebbles, summarized in Table 5.1. It will, however, be remembered that $E$ for chert is about 2·8 to 3·0 times that for dolomite, and it will also be noticed that $3^{1/3} \times 38 = 55$ which is in fair agreement with equation (5.15). This will at once suggest that the curves of Fig. 5.10 will be brought into even better agreement if the quantity $d^2/Db$ is multiplied by $(E)^{1/3}$. However, since in the present work chert is taken as of unit hardness, this result is best expressed as $(d^2/Db)(E_c/E)^{1/3}$, where $E_c$ is the value of $E$ for chert. This quantity is then used as the complete independent variable.

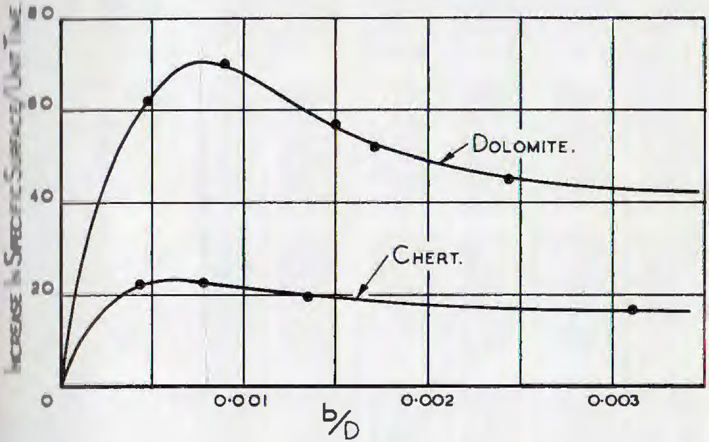


FIG. 5.10

$\frac{ds}{dt} = S$ $\frac{ds}{dE} = k \cdot S = k \cdot \frac{1}{d}$

TABLE 5.2

2 DOLOMITE

| Feed mesh | Ball size d (in.) | Mean feed size (in.) | Mean product size (in.) | b Mean particle size (in.) | $\frac{b}{d}$ | $\frac{d}{D}$ | $\frac{b}{D}$ | Increase in surface per minute |
|---|---|---|---|---|---|---|---|---|
| 65 | 2·75 | 0·0098 | 0·0089 | 0·0094 | 0·0034 | 0·145 | 0·00050 | 25·6 |
| | 2·0 | 0·0098 | 0·0085 | 0·0092 | 0·0046 | 0·105 | 0·00048 | 36·0 |
| | 1·38 | 0·0098 | 0·0079 | 0·0089 | 0·0065 | 0·072 | 0·00047 | 42·5 |
| | 1·0 | 0·0098 | 0·0072 | 0·0085 | 0·0085 | 0·052 | 0·00045 | 50·0 |
| | 0·75 | 0·0098 | 0·0071 | 0·0085 | 0·0113 | 0·039 | 0·00045 | 57·5 |
| ⟵→ | 0·62 | 0·0098 | 0·0070 | 0·0084 | 0·0138 | 0·032 | 0·00045 | 60·6 |
| 35 | 2·75 | 0·0210 | 0·0140 | 0·0175 | 0·0063 | 0·145 | 0·00093 | 35·0 |
| | 2·0 | 0·0210 | 0·0138 | 0·0174 | 0·0087 | 0·105 | 0·00092 | 43·8 |
| | 1·38 | 0·0210 | 0·0138 | 0·0174 | 0·0127 | 0·072 | 0·00092 | 57·0 |
| | 1·0 | 0·0210 | 0·0134 | 0·0172 | 0·0172 | 0·052 | 0·00092 | 67·5 |
| → | 0·75 | 0·0210 | 0·0126 | 0·0168 | 0·0225 | 0·039 | 0·00089 | 70·0 |
| | 0·62 | 0·0210 | 0·0146 | 0·0178 | 0·0285 | 0·032 | 0·00094 | 59·6 |
| 20 | 2·75 | 0·0400 | 0·0156 | 0·0278 | 0·0101 | 0·145 | 0·00147 | 35·8 |
| | 2·0 | 0·0400 | 0·0119 | 0·0258 | 0·0129 | 0·105 | 0·00135 | 47·0 |
| | 1·38 | 0·0400 | 0·0126 | 0·0263 | 0·0191 | 0·072 | 0·00139 | 54·5 |
| → | 1·0 | 0·0400 | 0·0109 | 0·0255 | 0·0255 | 0·052 | 0·00134 | 57·5 |
| | 0·75 | 0·0400 | 0·0145 | 0·0272 | 0·0362 | 0·039 | 0·00144 | 51·6 |
| | 0·62 | 0·0400 | 0·0301 | 0·0350 | 0·0565 | 0·032 | 0·00185 | 30·0 |
| 14 | 2·75 | 0·0480 | 0·0152 | 0·0316 | 0·0115 | 0·145 | 0·00167 | 38·0 |
| | 2·0 | 0·0480 | 0·0151 | 0·0315 | 0·0158 | 0·105 | 0·00167 | 47·0 |
| | 1·38 | 0·0480 | 0·0186 | 0·0333 | 0·0240 | 0·072 | 0·00175 | 47·5 |
| ⟵→ | 1·0 | 0·0480 | 0·0166 | 0·0326 | 0·0326 | 0·052 | 0·00171 | 50·0 |
| | 0·75 | 0·0480 | 0·0329 | 0·0405 | 0·0540 | 1·039 | 0·00210 | 40·0 |
| 10 | 2·75 | 0·0690 | 0·0226 | 0·0406 | 0·0167 | 0·145 | 0·00240 | 33·8 |
| | 2·0 | 0·0690 | 0·0232 | 0·0460 | 0·0230 | 0·105 | 0·00240 | 37·6 |
| → | 1·38 | 0·0690 | 0·0230 | 0·0460 | 0·0330 | 0·072 | 0·00240 | 45·0 |
| | 1·0 | 0·0690 | 0·0286 | 0·0488 | 0·0488 | 0·052 | 0·00258 | 33·2 |

$\frac{0.021}{0.098}$

$\frac{1}{0.021} = 47.5$

$\frac{1}{0.040} = 25.0$

$\frac{1}{0.048} = 20.8$

$\frac{1}{0.069} = 14.5$

Thus, the functional relationships $(d/D)$ and $(b/D)$ may be summarized into the two curves of Fig. 5.11 and Fig. 5.12; where Fig. 5.11 consists of the curves of Fig. 5.10 brought into coincidence by multiplying the ordinates of the curve for chert by 2·8 and then plotting the resulting curve on a scale such that the vertex gives 100 %, and

5.12

b = particle diameter
d = Ball
D = Mill

3.8

CHERT

| Feed mesh | Ball size (in.) | Mean feed size (in.) | Mean product size (in.) | Mean particle size (in.) | $\frac{b}{d}$ | $\frac{d}{D}$ | $\frac{b}{D}$ | Increase in surface per minute |
|---|---|---|---|---|---|---|---|---|
| 65 | 2·75 | 0·0098 | 0·0075 | 0·0086 | 0·0031 | 0·145 | 0·000455 | 11·1 |
| | 2·0 | 0·0098 | 0·0075 | 0·0086 | 0·0043 | 0·105 | 0·000455 | 15·0 |
| | 1·38 | 0·0098 | 0·0071 | 0·0084 | 0·0061 | 0·072 | 0·000440 | 19·5 |
| | 1·0 | 0·0098 | 0·0070 | 0·0084 | 0·0084 | 0·052 | 0·000440 | 21·4 |
| | 0·75 | 0·0098 | 0·0070 | 0·0084 | 0·0113 | 0·039 | 0·000440 | 23·3 |
| | 0·62 | 0·0098 | 0·0074 | 0·0086 | 0·0139 | 0·032 | 0·000455 | 20·6 |
| 35 | 2·75 | 0·0184 | 0·0122 | 0·0153 | 0·0056 | 0·145 | 0·00081 | 15·1 |
| | 2·0 | 0·0184 | 0·0116 | 0·0150 | 0·0075 | 0·105 | 0·00078 | 18·7 |
| | 1·38 | 0·0184 | 0·0108 | 0·0146 | 0·0106 | 0·072 | 0·00077 | 20·9 |
| | 1·0 | 0·0184 | 0·0109 | 0·0146 | 0·0146 | 0·052 | 0·00077 | 23·0 |
| | 0·75 | 0·0184 | 0·0107 | 0·0145 | 0·0193 | 0·039 | 0·00076 | 22·7 |
| | 0·62 | 0·0184 | 0·0132 | 0·0156 | 0·0252 | 0·032 | 0·00082 | 18·3 |
| 20 | 2·75 | 0·0374 | 0·0142 | 0·0258 | 0·0094 | 0·145 | 0·00134 | 14·4 |
| | 2·0 | 0·0374 | 0·0138 | 0·0256 | 0·0128 | 0·105 | 0·00135 | 16·4 |
| | 1·38 | 0·0374 | 0·0138 | 0·0256 | 0·0185 | 0·072 | 0·00135 | 19·3 |
| | 1·0 | 0·0374 | 0·0142 | 0·0258 | 0·0258 | 0·052 | 0·00136 | 18·4 |
| | 0·75 | 0·0374 | 0·0174 | 0·0274 | 0·0365 | 0·039 | 0·00145 | 17·4 |
| | 0·62 | 0·0374 | 0·0285 | 0·0329 | 0·0530 | 0·032 | 0·00171 | 12·1 |
| 10 | 2·75 | 0·0700 | 0·0328 | 0·0514 | 0·0187 | 0·145 | 0·00272 | 13·4 |
| | 2·0 | 0·0700 | 0·0390 | 0·0545 | 0·0272 | 0·105 | 0·00286 | 15·3 |
| | 1·38 | 0·0700 | 0·0512 | 0·0606 | 0·0441 | 0·072 | 0·00320 | 15·7 |
| | 1·0 | 0·0700 | 0·0530 | 0·0615 | 0·0615 | 0·052 | 0·00325 | 15·7 |
| | 0·75 | 0·0700 | 0·0622 | 0·0661 | 0·0885 | 0·029 | 0·00350 | 11·4 |

Fig. 5.12 is the resulting curve when the data of Fig. 5.9 are brought into coincidence by the use of the curve of Fig. 5.11, the resulting curve being plotted on a percentage scale.

Consider now the form of the curve Fig. 5.12. For a given size mill and particle, but with an extremely small ball, the rate of grinding will be low; since the ball will not contain sufficient energy to produce much new surface. With increasing ball size, however, the rate of grinding

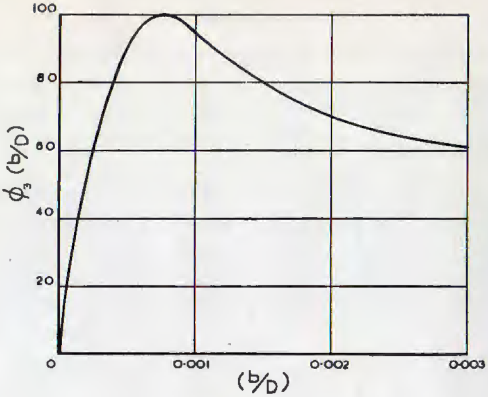


Fig. 5.11

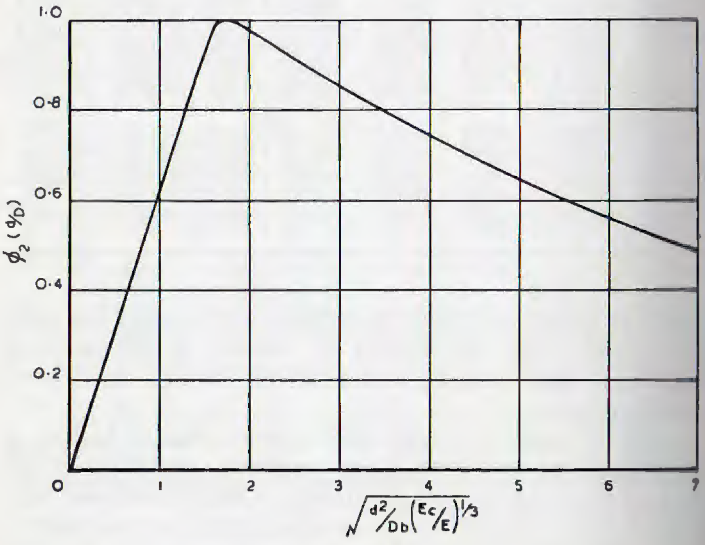


Fig. 5.12

will increase since the increased energy will bring about increased surface production per blow. This gain is in part offset by the decreasing number of balls, and so blows, with increasing ball size but, nevertheless, the result is an increase in rate of grinding. When the balls have reached a size such that there is sufficient energy to crush practically any particle which presents itself then, theoretically, the rate of grinding would remain constant since the energy produced by an impact of a ball is proportional to the cube of the ball diameter, $d^3$, whilst the number of impacts, per unit time, varies as the number of balls, that is, varies inversely as the cube of the ball diameter, that is, directly as $1/d^3$.

Thus

$$
\begin{aligned}
S/t &\propto d^3(1/d^3) \\
&= \text{constant} \qquad \ldots\ldots \quad (5.17)
\end{aligned}
$$

When the balls are excessively large, however, the rate of grinding decreases since, as explained in Chapter 4, the production of surface per unit of energy decreases with very high energy densities and also the efficiency of the blow is impaired when the balls are unduly large in relation to the mill size. Thus, the falling branch of the curve is obtained. (In fact this reduction is over-emphasized by the use of the root scale; on a natural scale the decrease is much more gradual.) Furthermore, as the ball diameter increases, the number of particles which may be trapped, at a point of contact, is increased and since an impact which contains enough energy to crush a single particle does not necessarily contain enough to crush two, and so on; this cause in itself can result in a reduced rate of grinding.

Thus several causes operate to reduce the rate of grinding with excessive ball size and this is exactly what has been found by experiment. It is interesting to note that the results of Gross, Fig. 5.7, are in close agreement with those deduced from the work of Coghill and Devaney, since a suitable change of the scale ratios will bring the

to impaire = ἐλαττώνω.

curve of Fig. 5.7 into extremely close agreement with that of Fig. 5.12.

Turning now to Fig. 5.11; if the ratio of $(d/D)$ is constant then in fact $(b/D)$ is directly proportional to $(b/d)$; the ratio of particle diameter to ball diameter. It follows from equation (5.3) and the explanation given earlier, however, that the number of particles which would be crushed between two balls is given by the expression

$$N \propto \sqrt{b} \qquad \ldots. \ (5.18)$$

when $b$ is small compared with $d$.

Thus, for the initial part of the curve, the rate of grinding would be expected to vary as the square root of the particle diameter and, in fact, this part of the curve of Fig. 5.11 coincides closely with such a relationship. As the particle size increases, however, the angle $\alpha$, Fig. 5.1, increases until, at a certain critical particle size the particle is expelled from between the balls; because the frictional forces are insufficient to hold it in place. Clearly, when this condition is reached the particles are expelled, instead of crushed, and the rate of grinding should, ideally, fall to zero. Since in practice, however, the particles have a range of sizes and the ball surfaces are not perfectly smooth, the curve shows a steady decrease in grinding rate, instead of a sudden drop to zero.

This view can, to some extent, be confirmed as follows. For the maximum, at which projection commences, $(b/d)=0{\cdot}00075$. Assuming this condition also coincides with the maximum of the curve of Fig. 5.12, $(d^2/Db)=2$, or $(d/D)^2=2\times0{\cdot}00075$. Thus $d/D=0{\cdot}04$ and so $(b/d)=0{\cdot}00075/0{\cdot}04\simeq0{\cdot}02$.

Thus, $\cos\alpha=d/1{\cdot}02d=0{\cdot}98$ or $\alpha=11^\circ\,30'$ approx. Also a study of the forces acting on the particle shows that the coefficient of friction, $\mu$, is given by $\mu=\tan\alpha$ or $\mu\simeq0{\cdot}2$; this value being quite as close to the value which would be expected as can reasonably be hoped.

The influence of the ratio of rod diameter to mill diameter on the rate of grinding for a rod mill may now well be considered, and in Fig. 5.13 are given curves of the rate of surface production related to $(d/D)$ ratio, based upon Table 11 of the work of Coghill and Devaney. Although the data on the point are limited, it appears that the rate of grinding increases steadily with the diameter of the rod and for large rods becomes sensibly constant. This result is not surprising, however, since the rods act as crushing rolls, and all the particles have to pass between the crushing surfaces. Thus, the probability of trapping a particle is not such an important variable as with balls as the grinding media. Clearly, thus, the rate of grinding will increase initially, after which the increased production per impact is offset by the reduced number of impacts per unit time; this leading to a constant rate of grinding, as suggested for the ball mill.

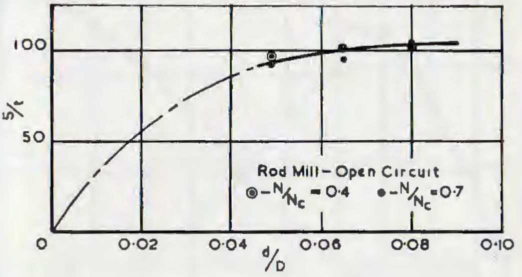


FIG. 5.13

In all cases the production of specific surface would be expected to be directly proportional to the time of grinding and so it follows that

$$\phi_5(gt^2/D) = (gt^2/D)^{1/2} \qquad \dots . \ (5.19)$$

Finally, there is the question of how the rate of grinding varies with the group $(g/DN^2)$; that is, from Chapter 3, with $(N/N_c)$.

In Fig. 5.14 is given the values of Rittinger's constant, at various speeds, as determined by Gross for batch milling a dry powder. From this curve it is seen that the value of Rittinger's constant, which is proportional to the rate of increase of surface of the powder, varies almost directly with the mill speed. Theoretically, the charge would centrifuge at $N/N_c = 1{\cdot}0$ and, since there can then be no grinding, the curve should theoretically drop to zero along a line coincident with the line $N/N_c = 1{\cdot}0$. In practice, since there is slip between the mill shell and charge, the charge does not centrifuge at $N/N_c = 1{\cdot}0$, but the operation takes place progressively over a range of speed. Thus, the curve would be expected to gradually fall to zero for speeds of rotation greater than the critical.

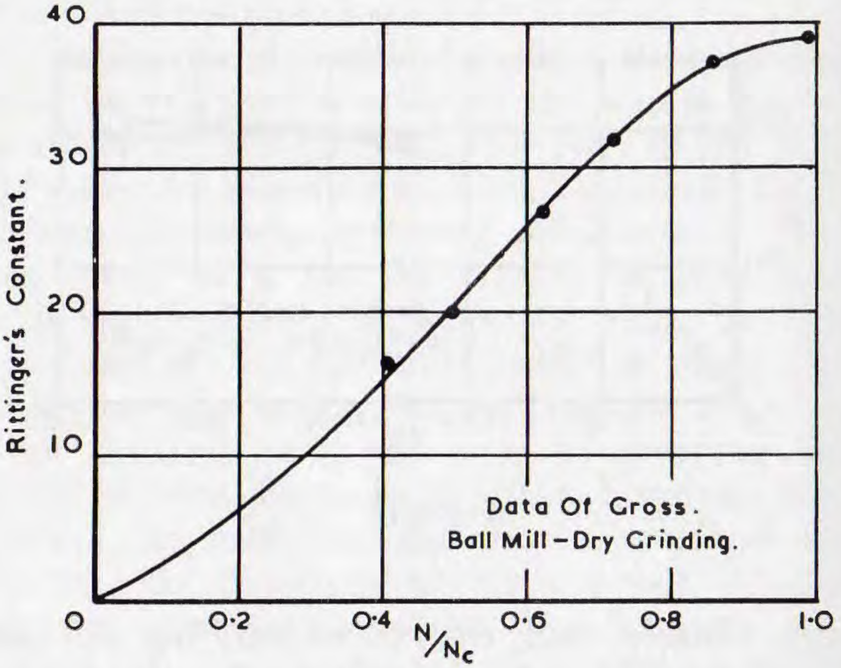


FIG. 5.14

In Fig. 5.15 are given the results of tests, by Coghill and Devaney, on a rod mill wet-grinding chert on open circuit. In this case the rate of grinding falls rapidly for speeds in excess of about 65 % of the critical; this result being in contrast with that for dry milling, just quoted, in which no such fall occurs. This decrease in the rate of grinding at high speeds of rotation, when wet-grinding, would be expected. Thus, for a very wet pulp or for a dry powder, there would be no appreciable cohesion of the powder charge and so centrifuging of the grinding media would be uninfluenced by the presence of the pulp and would occur at, practically, the critical speed.

A moderately dry pulp would, however, exhibit considerable cohesive strength and so the balls would tend to adhere to the mill shell. For very cohesive pulps, the adhesion might be sufficient to allow a considerable portion of the charge to be carried around with the shell, even when the actual speed of rotation is considerably below the critical. This explanation is sufficient to account completely for the observed results. In fact this phenomenon is frequently observed in ball mills used to mix the pigment into the liquid vehicle in the manufacture of paint. Similar results for tests on ball mills are shown in Fig. 5.16, and it is seen that curves of the type shown in Fig. 5.15 fit the experimental points well.

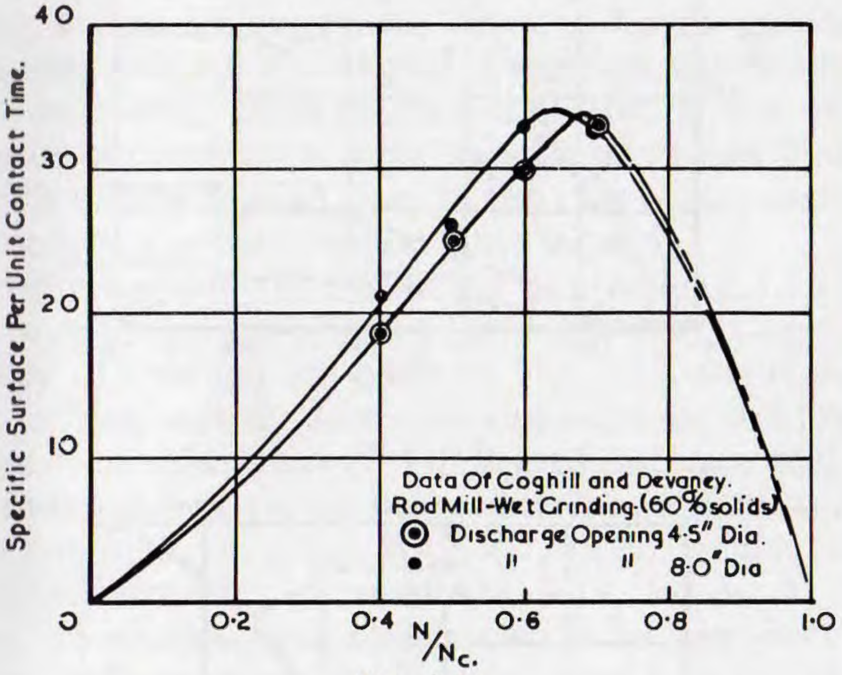


FIG. 5.15

These results can be summarized in a curve of the form of Fig. 5.17; this curve giving $\phi_4(N/N_c)$ in terms of $(N/N_c)$ and the pulp composition. Unfortunately there are

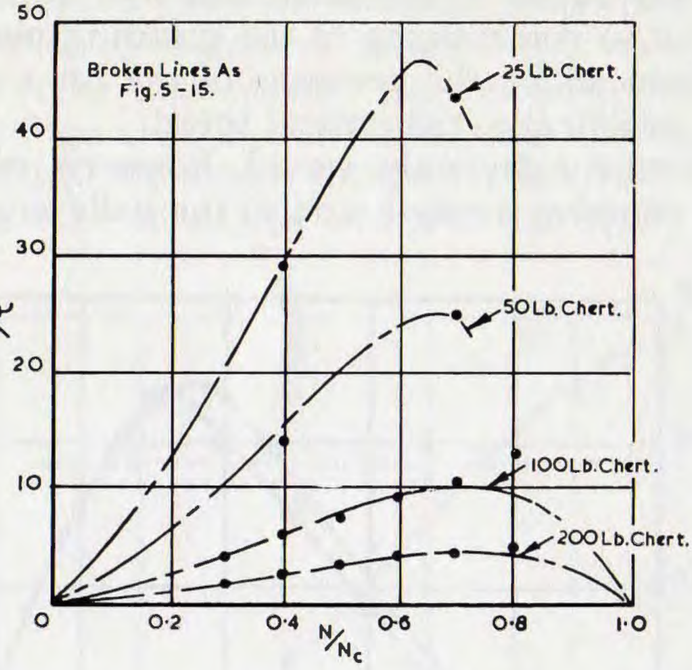


FIG. 5.16

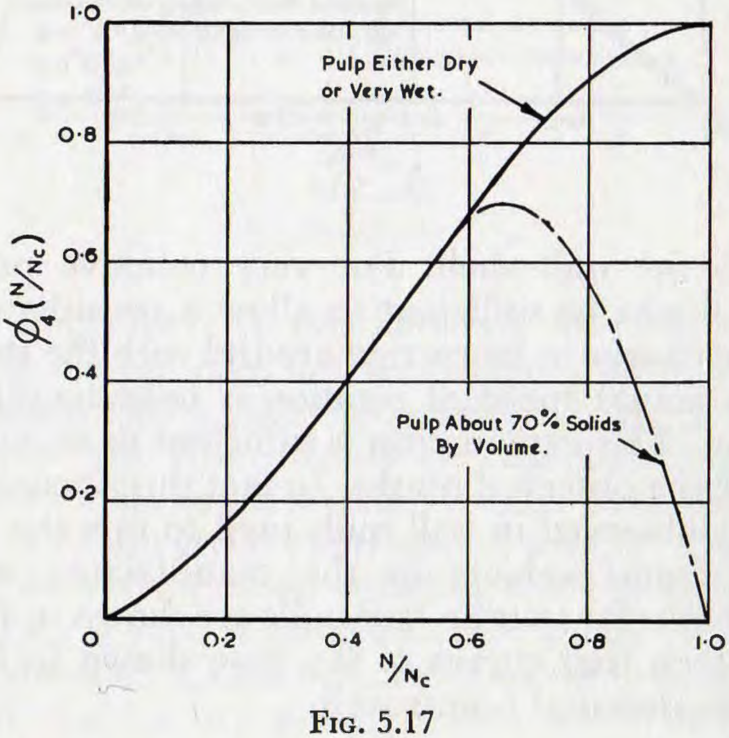


FIG. 5.17

insufficient data for the effect of pulp composition to be fully evaluated but linear interpolation between the curves of Fig. 5.17 is probably sufficient for most purposes; especially as but few mills run at much above about 75 % of the critical speed.

It will be noticed that in Fig. 5.15 the increase of specific surface *per unit contact time* is plotted for the continuous circuit operation and that when plotted on this basis the curves correspond with those for batch mills. Thus it follows that if the surface produced per unit contact time is used, a direct comparison between batch milling and continuous milling is obtained. Doubtless this is the basis for the statement, often made without proof, that the performance of continuous mills may be predicted from tests on batch mills provided that, in the case of the continuous operation, the actual contact time is used.

Considering now the mill filling parameter $\phi_7(J)$, results based upon tests by Gross (Table 32) and Coghill and Devaney (Table 19) are given in Fig. 5.18, the former for batch milling and the latter for open-circuit milling. It is at once seen that a simple change of ordinate will bring these independent curves into coincidence; and, since the total surface, $S_t$, is given by $S \times J$, these results may be generalized into the curve of Fig. 5.19, for $\phi_7(J)$. These curves are of the form which would be expected since, with no balls present ($J=0$) there would be no appreciable grinding. Similarly, when the mill is packed full with balls ($J=100\%$) there can be no ball motion and so no grinding. In between these values the rate of grinding must attain a maximum. Furthermore, for small ball fillings the rate of grinding increases almost directly with the ball charge and this would be expected since the probability of impact increases with the number of balls.*

The next variable which may well be studied is the effect of the variation of the fraction of the void space

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* A ball can only collide with its immediate neighbours, and so, per unit time, the probability of collision is *not* the probability of collision of a given ball with every other ball in the mill.

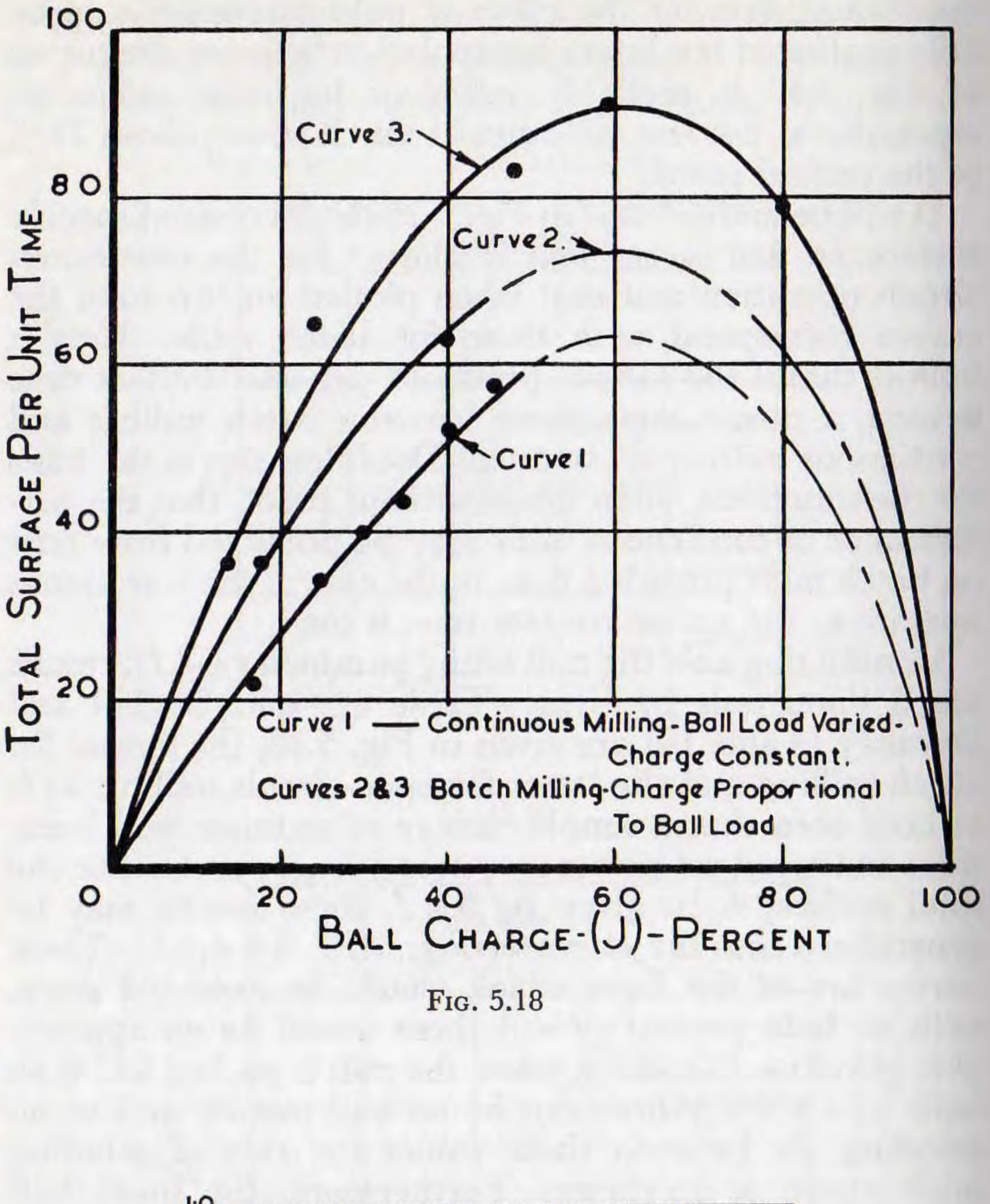

FIG. 5.18

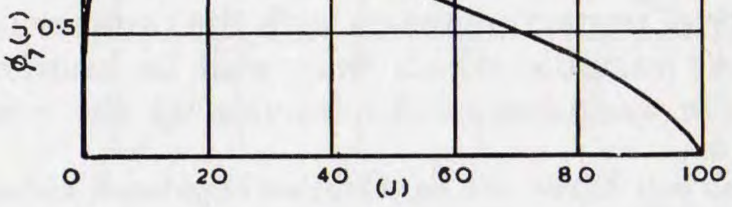

FIG. 5.19

between the balls which is occupied by solid particles, that is the function $\phi_9(V)$. In Fig. 5.20 are plotted the results of a number of tests by Coghill and Devaney (Tables 9, 13 and 14) for ball and rod mills grinding various materials at various speeds; the results being correlated by the elimination of the effects of critical speed by the use of a multiplicative factor, and again the agreement is satisfactory.

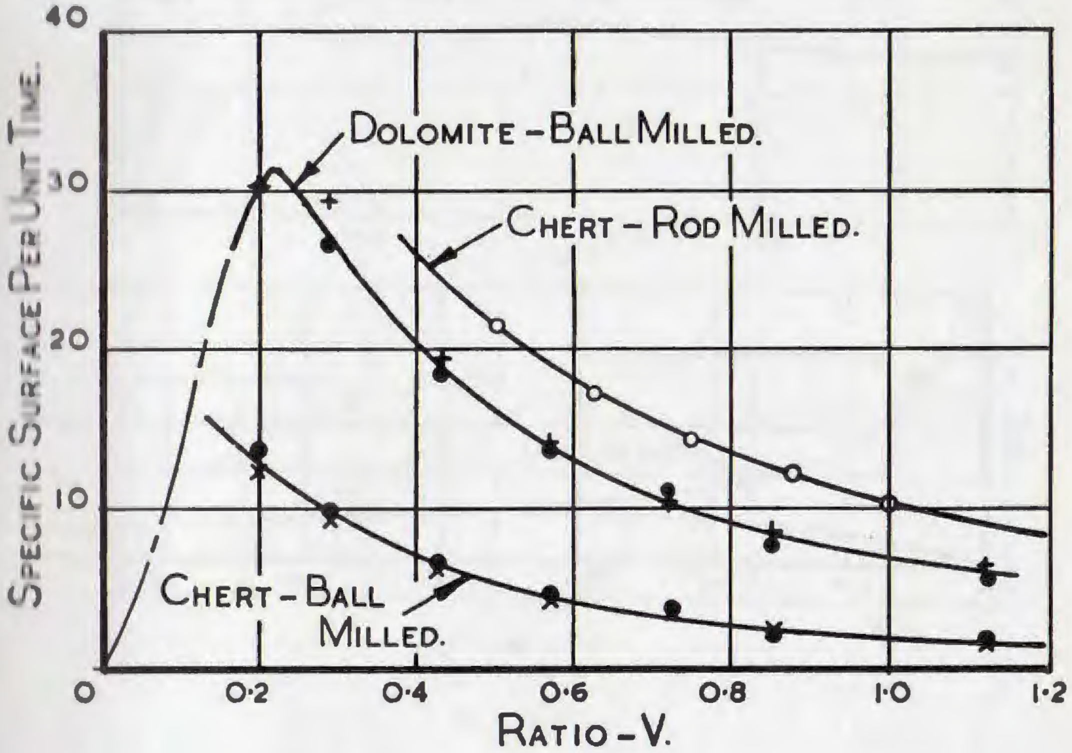

Fig. 5.20

When these results are plotted on a basis comparable with the theoretical curves as deduced from the theory of Rose, explained earlier, Fig. 5.21a, it is seen that the experimental curves have the same form as the theoretical ones, shown in Fig. 5.21b. The theoretical curves are calculated for materials of zero and infinite hardness, and the curves for real materials show intermediate characteristics, as would be expected. Furthermore, when curves of total surface change per unit time, Fig. 5.22a, are plotted

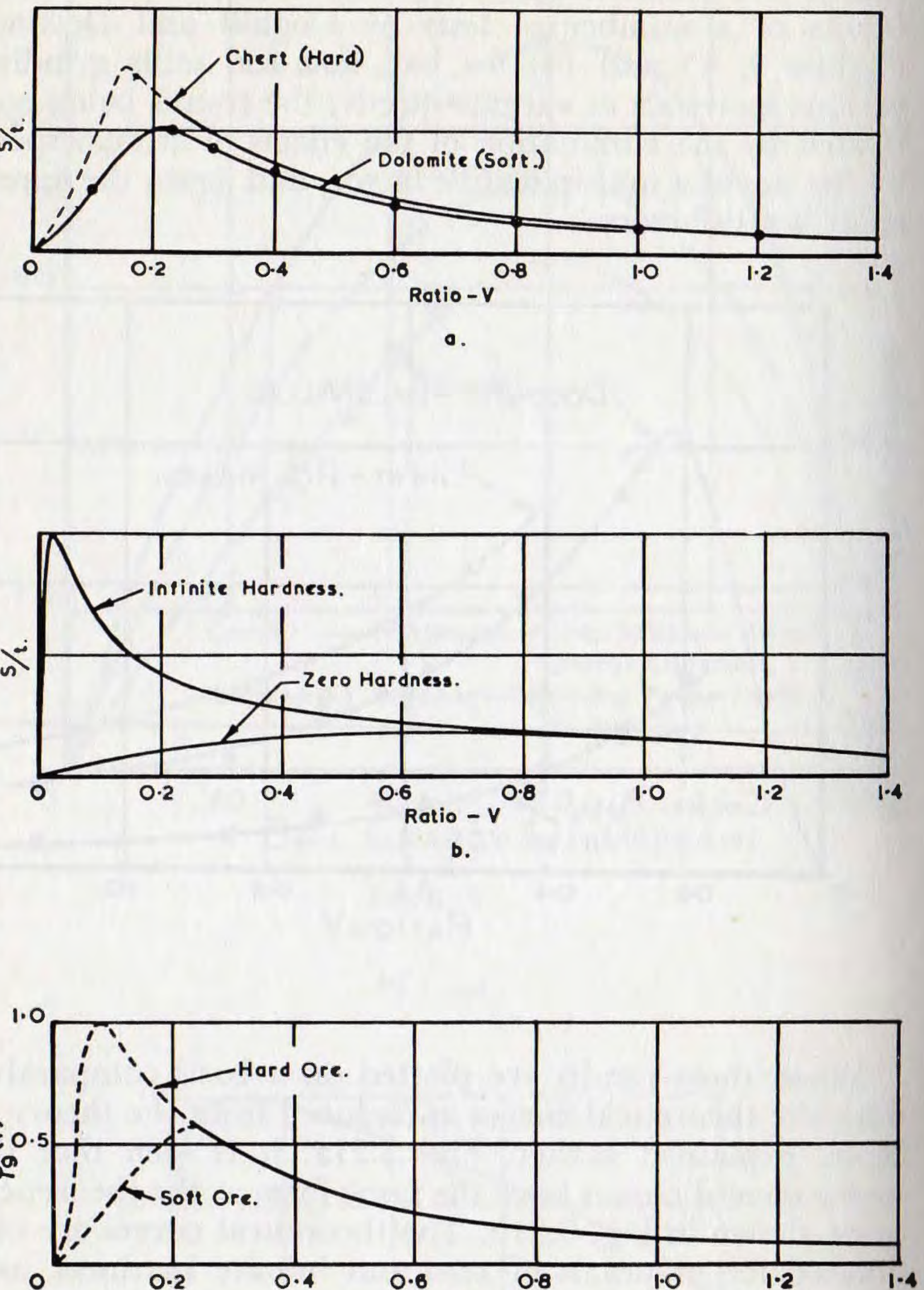
Chert (Hard)
Dolomite (Soft.)
S/t.
0
0·2
0·4
0·6
0·8
1·0
1·2
1·4
Ratio – V
a.
Infinite Hardness.
Zero Hardness.
S/t.
0
0·2
0·4
0·6
0·8
1·0
1·2
1·4
Ratio – V
b.
1·0
Hard Ore.
$\phi_g$ (V) 0·5
Soft Ore.
0
0·2
0·4
0·6
0·8
1·0
1·2
1·4
Ratio – V

FIG. 5.21

and compared with those calculated theoretically, Fig. 5.22b, the agreement in form is again seen to be reasonably good.

Thus these results support the general validity of the theoretical treatment proposed by Rose; this treatment being, as far as is known, the only one so far proposed by which the effect of the mill parameters has been deduced from theoretical considerations. Until a theory is sufficiently developed, however, to allow the calculation of this function from the basic physical data, the curve of Fig. 5.21c probably gives the value of $\phi_9(V)$ with an accuracy sufficient for design purposes.

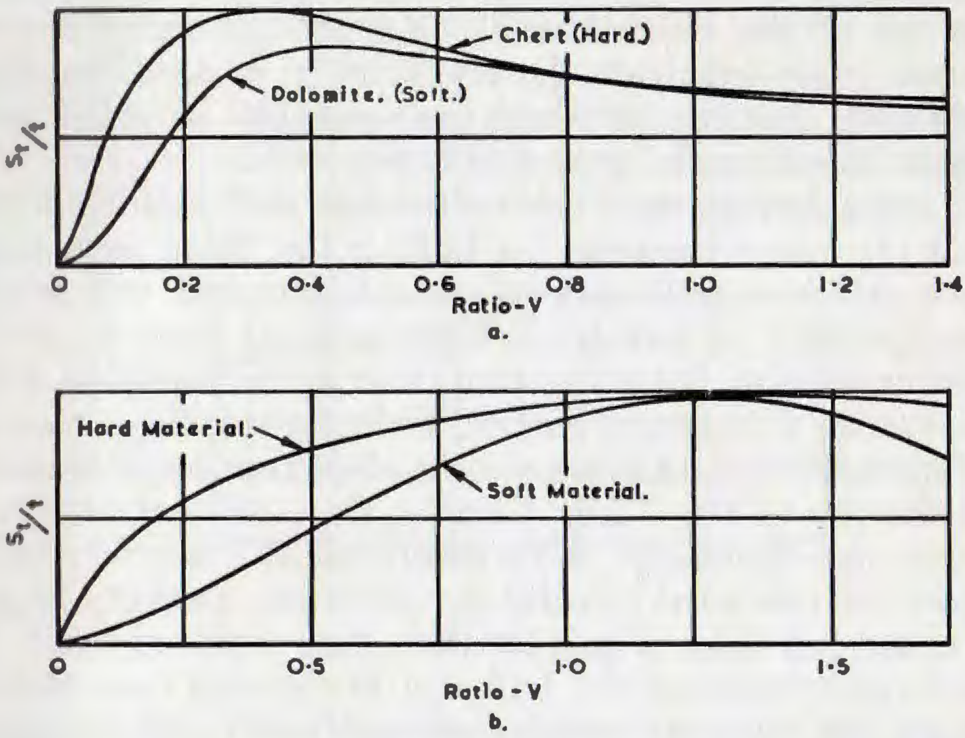

FIG. 5.22

The question of the effect of the consistency of the pulp $\phi_8(U)$ would appear to be, at least as a first approximation, one which is fairly easily treated. However, the published data relating to this variable are in fact so conflicting as to render impossible the drawing of firm conclusions.

When grinding a pulp, a mill may, ideally, be operated in accordance with either of two limiting cases; one in which the *pulp* just fills the space between the balls and the other in which the *solid material in the pulp* just fills that space. In addition there are an unlimited number of other possibilities, in which neither of these conditions are observed, and it is probably the great number of possible conditions of operation which lead to the inconclusive nature of the experimental evidence on this variable. From other evidence, however, it is possible to draw tentative conclusions which are possibly of value until sound experimental evidence is available.

In the first place, if the mill is operated in such a way that the space between the balls is just filled with pulp, then clearly a reduction in the solid content will reduce the quantity of powder present and, at least to a first approximation, have the same effect as the variation of the parameter $V$ discussed earlier. Thus, for this case, it would be expected that an approximate allowance could be made by the use of Fig. 5.21c. Clearly, since this calculation is based on a knowledge of the volume of *solid material* present it can be extended to cases in which the quantity of *pulp* is either less than or greater than that required to fill the void space between the balls. In the second case, that in which the solid material is just sufficient to fill the space between the balls, it would be expected that the consistency of the pulp would have but little effect and, as a first approximation, may be neglected.

In both cases the fluidity of the pulp will have some effect, in that, with a more fluid pulp the finer particles will be washed from between the balls and the efficiency of the crushing process improved. This action will also lead to an amount of selective grinding, but this subject will be treated in a later chapter. Thus, an approximation to the value of $\phi_8(U)$ may be made.

The diameter, $u$, of the exit aperture from a mill would be expected to have but little effect upon the rate of milling since, other things being equal, variations in this dimension will merely vary the flow rate through the mill. In addition the effects of variation of this quantity are included when the calculations are based on the actual contact time of the material with the mill. There might remain, however, some small effects arising from variation in the perfection of the mixing in the mill, with varying exit diameters, and similar minor influences; but such factors would be expected to be unimportant. The experimental data relating to this variable are sparse, but the curve of Fig. 5.23 indicates the trends as far as they can be deduced from the published evidence. It is at once seen that the effect of this variable is small. It is clear that the length of the mill is also a significant variable in this respect; since this affects the angle of slope of the surface of the charge, but there are insufficient data for the relationships to be evaluated.

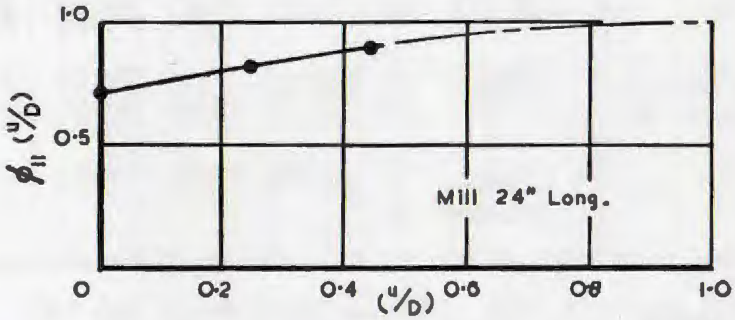

FIG. 5.23

The question of the influence of the ratio of new feed to circulating load does not appear to have received systematic treatment, but general conclusions based on the previous treatment may be drawn.

From the previous treatment it is clear that, provided the ratio of ball size to average particle size is not too small, the rate of grinding is not greatly influenced by the size distributions of the granular material, and the foregoing work may be used for the estimation of the increase of surface in the mill. Thus, the combined feed may be treated as a normal feed and the product size calculated

TABLE

| Authority | Material | Mill size Diam. (in.) × Length | $E$ | $\frac{N}{N_c}$ | $U$ | $\phi_4\left(\frac{N}{N_c}\right)$ |
|---|---|---|---|---|---|---|
| Coghill and Devaney* (Table 25) | Chert | $19\times36$ | 1·0 | 0·60 | 0·60 | 0·68 |
| | ,, | $24\times24$ | 1·0 | 0·60 | 0·60 | 0·68 |
| | Dolomite | $68\times48$ | 0·36 | 0·80 | 0·77 | 0·65 |
| | ,, | $19\times36$ | 0·36 | 0·80 | 0·77 | 0·65 |
| Gow, Guggenheim, Coghill† (Table 16) | Dolomite | $78\times144$ | 0·36 | 0·715 | 0·78 | 0·70 |
| Maxson, Cadena and Bond‡ | Quartz 6–28 | $12\times12$ | 1·08 | 0·89 | 1·00 | 0·98 |
| | 6–48 | | 1·0 | 0·89 | 1·00 | 0·98 |
| | 6–100 | | 1·0 | 0·89 | 1·00 | 0·98 |
| | 6–200 | | 1·0 | 0·89 | 1·00 | 0·98 |
| Coghill and Devaney, Rod Mill (Table 9) | Chert | $19\times36$ | 1·0 | 0·40 | 0·60 | 0·40 |
| | ,, | $19\times36$ | 1·0 | 0·70 | 0·60 | 0·70 |
| Starke§ | Cement Clinker | $36\times48$ | 0·5 | 0·75 | 1·00 | 0·87 |

* *Ball Mill Grinding*. U.S. Bur. of Mines Tech. Paper, No. 581.

† *Trans. Amer. Inst. min. (metall.) Engrs*, 1934, **112**, 24.

‡ *Ibid.*, **112**, 130.

§ *Rock Prod.*, 1935, **38**, 40.

from it by the use of the various functional relationships already discussed.

On the basis of the present analysis equation (5.2) for the computation of the change of specific surface, may be simplified.

It has been suggested that $\phi_1(D^2g\varrho/E) = (D^2g\varrho/E)$ and $\phi_5(gt^2D) = (gt^2/D)^{1/2}$, so it follows that

$$S = K\left(\frac{D^{0\cdot5}\varrho t}{E}\right)\cdot\phi_2\left(\frac{d}{D}\right)\cdot\phi_3\left(\frac{b}{D}\right)\cdot\phi_4\left(\frac{N}{N_c}\right)\cdot\phi_6\left(\frac{\sigma}{\varrho}\right)\cdot\phi_7(J)$$
$$\times\phi_9(V)\cdot\phi_{11}\left(\frac{u}{D}\right) \quad \ldots\ldots \quad (5.20)$$

5.3

| $\left(\frac{d}{D}\right)$ | $\left(\frac{b}{D}\right)$ | $\phi_2\left(\frac{d}{D}\right)$ | $\phi_3\left(\frac{b}{D}\right)$ | $(V)$ | $\phi_9(V)$ | $(J)$ | $\phi_7(J)$ | $\left(\frac{\varrho}{\sigma}\right)$ | $\phi_6\left(\frac{\sigma}{\varrho}\right)$ | $K$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0·05 | 0·0010 | 1·0 | 0·95 | 1·00 | 1·00 | 0·50 | 0·70 | 3·0 | 0·6 | 0·036 |
| 0·05 | 0·0008 | 1·0 | 1·00 | 1·00 | 1·00 | 0·50 | 0·70 | 3·0 | 0·6 | 0·030 |
| 0·05 | 0·0003 | 0·75 | 0·73 | 1·00 | 1·00 | 0·50 | 0·70 | 3·0 | 0·6 | 0·006 |
| 0·05 | 0·0010 | 0·97 | 0·95 | 1·00 | 1·00 | 0·50 | 0·70 | 3·0 | 0·6 | 0·030 |
| 0·03 | 0·00013 | 0·83 | 0·50 | 1·00 | 1·0 | 0·47 | 0·72 | 3·0 | 0·6 | 0·0082 |
| | | | | — | | | | | | |
| 0·094 | 0·005 | 0·85 | 0·60 | 0·36 | 4·0 | 0·213 | 0·96 | 3·0 | 0·6 | 0·0079 |
| 0·094 | 0·005 | 0·85 | 0·60 | 0·36 | 4·0 | 0·213 | 0·96 | 3·0 | 0·6 | 0·0104 |
| 0·094 | 0·005 | 0·85 | 0·60 | 0·36 | 4·0 | 0·213 | 0·96 | 3·0 | 0·6 | 0·0121 |
| 0·094 | 0·005 | 0·85 | 0·60 | 0·36 | 4·0 | 0·213 | 0·96 | 3·0 | 0·6 | 0·0144 |
| 0·066 | 0·0007 | 0·90 | 1·0 | 1·0 | 1·0 | 0·45 | 0·75 | 3·0 | 0·6 | 0·78 |
| 0·066 | 0·0007 | 0·90 | 1·0 | 1·0 | 1·0 | 0·45 | 0·75 | 3·0 | 0·6 | 0·74 |
| 0·028 | 0·001 | 1·00 | 0·60 | 0·3 | 5·0 | 0·48 | 0·7 | 2·0 | 0·8 | 0·012 |

where $\phi_2(d/D)$ is read from Fig. 5.12,
$\phi_3(b/D)$ is read from Fig. 5.11,
$\phi_4(N/N_c)$ is read from Fig. 5.17,
$\phi_6(\sigma/\varrho)$ is read from Fig. 5.5,
$\phi_7(J)$ is read from Fig. 5.19,
$\phi_9(V)$ is read from Fig. 5.21c,
and $\phi_{11}(u/D)$ is read from Fig. 5.23,

the graphs $\phi_8(U)$ and $\phi_{10}(N)$ being omitted since it has been shown that their effects are largely included by the correct computation of the values of other variables.

The value of the constant $K$ must be determined by reference to the results of experiment and there is an amount of data available for this purpose. In Table 5.3 are given a number of values; based on C.G.S. units, but with $E$ taken as unity for chert. All these figures are seen to be of the same order, there being about a 3 to 1 ratio between the extreme values. This scatter probably arises from difference in the bases of calculation of the change in specific surface; since the various workers have different methods. The agreement is such, however, as to suggest that the general method of approach is correct and that an accurate series of experiments for the evaluation of the constant is the primary requirement. On the basis of these limited data, however, it appears safe to draw the provisional conclusion that the value of the constant $K$, in equation (5.20), may be taken as

$$\left.\begin{aligned} K &= 0{\cdot}02 \quad \text{for a ball mill} \\ K &= 0{\cdot}75 \quad \text{for a rod mill} \end{aligned}\right\} \quad \ldots\ldots \quad (5.21)$$

with a deviation of, say, ±50%, but these figures will probably be modified as more data become available. Thus, the rate of change of specific surface may be calculated from the mill dimensions; provided the grindability of the material, relative to chert, is known.

There now remains the question of the grinding of a mixed material; this case being practically important since, for example, any ore contains at least two components. The problem has been studied by Holmes and Patching,(8) and some interesting tentative conclusions have been reached.

The rate of grinding is written in the form:

$$M = \frac{kW}{\xi} \quad \ldots\ldots \quad (5.22)$$

where $M$ is the weight of product per revolution, $\xi$ is the bulk density of the material, $W$ the weight of material in the mill and $k$ is the specific rate of reduction, and it is shown that the ratios $(k_1'/k_2')$ and $(k_1/k_2)$ are approximately equal and sensibly independent of the proportions of the two components in the mill; $k_1'$ and $k_2'$ being the specific rates of reduction for the two materials milled separately and $k_1$ and $k_2$ for them milled together.

Thus, it appears reasonable to conclude that when two materials are ground in combination, the rate of production of surface in each of the components is in the same ratio as it would be if the materials were milled separately. Furthermore, the rate of change of specific surface in each component is the same as when the component is milled separately. This view is also confirmed by Gow, Guggenheim, Campbell and Coghill, who show that for tests on chert-dolomite mixtures, the rate of size reduction is, very roughly, proportional to the grindabilities of the chert and dolomite when milled separately.

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## CHAPTER 6

# OTHER CONSIDERATIONS RELATING TO THE CHARACTERISTICS OF A MILL

IN the previous chapter the influence of the various physical quantities defining the mill and mill charge has been studied in connection with the performance of a mill as a device for the creation of new surface in the powder. For some purposes, however, it is also desirable that the product should have a preferred form of size distribution curve. For example, in mineral dressing it is usually necessary that the material below about 200 mesh should be a minimum. To some extent such modifications in the size distribution characteristics can be brought about by changes of the mill dimensions. It must be emphasized, however, that such changes are of limited scope but, even so, are often worth-while.

The first relationship to be considered is the effect of the variation of the ball diameter, or rod diameter, upon the size distribution obtained. In Fig. 6.1 are shown typical size frequency curves obtained by ball milling; these curves being deduced from the data of Coghill and Devaney(1).

As would be expected, the grinding of a coarse feed with small balls does not give rise to the effective grinding of the coarse particles in the feed but does produce a considerable amount of fine product. For the largest ball sizes, however, rapid grinding of the coarse fraction is attained. Turning attention to the finer feed, however, it will be observed that the largest ball size gives a coarser product than does the next size smaller balls. Thus it appears that

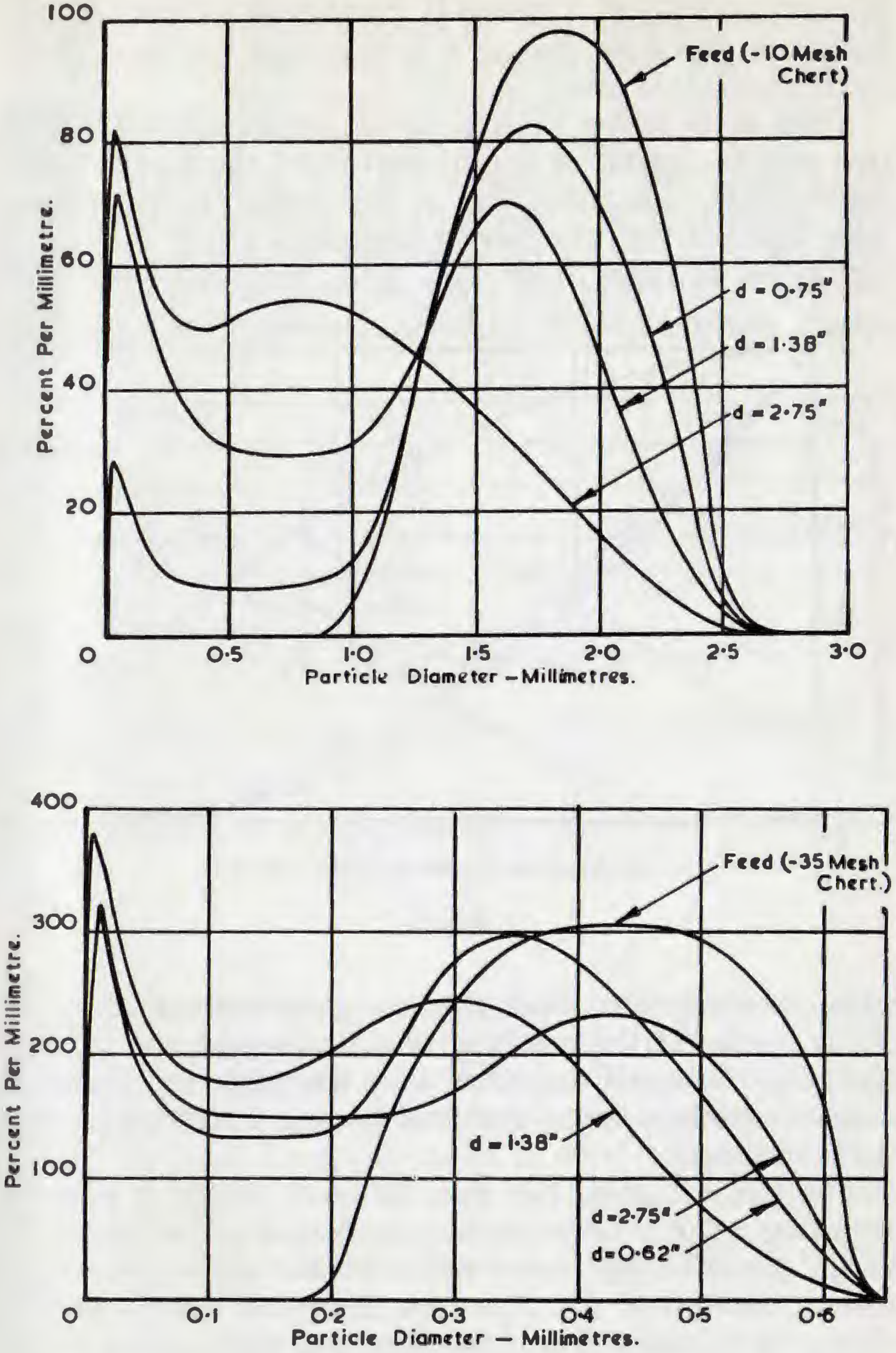
100
80
60
40
20
0
Percent Per Millimetre.
0·5
1·0
1·5
2·0
2·5
3·0
Particle Diameter – Millimetres.
Feed (-10 Mesh Chert)
d = 0·75"
d = 1·38"
d = 2·75"
400
300
200
100
0
Percent Per Millimetre.
0·1
0·2
0·3
0·4
0·5
0·6
Particle Diameter — Millimetres.
Feed (-35 Mesh Chert.)
d = 1·38"
d=2·75"
d=0·62"

FIG. 6.1

the coarser fractions may be ground, without the production of excessive fine material, by the use of balls of large diameter; however, the particle size must not be excessive or this benefit is lost.

From these rather limited data it would appear that the best selective grinding is obtained when the ball diameter is about 100–150 times that of the particle to be crushed. This figure is very roughly in accordance with that which would be deduced from Fig. 5.12, but close agreement cannot be expected since the two cases are not identical. Thus one is for the crushing of a powder having a single size of particle and the other is for the most rapid production of surface when a material having a wide size-range is comminuted.

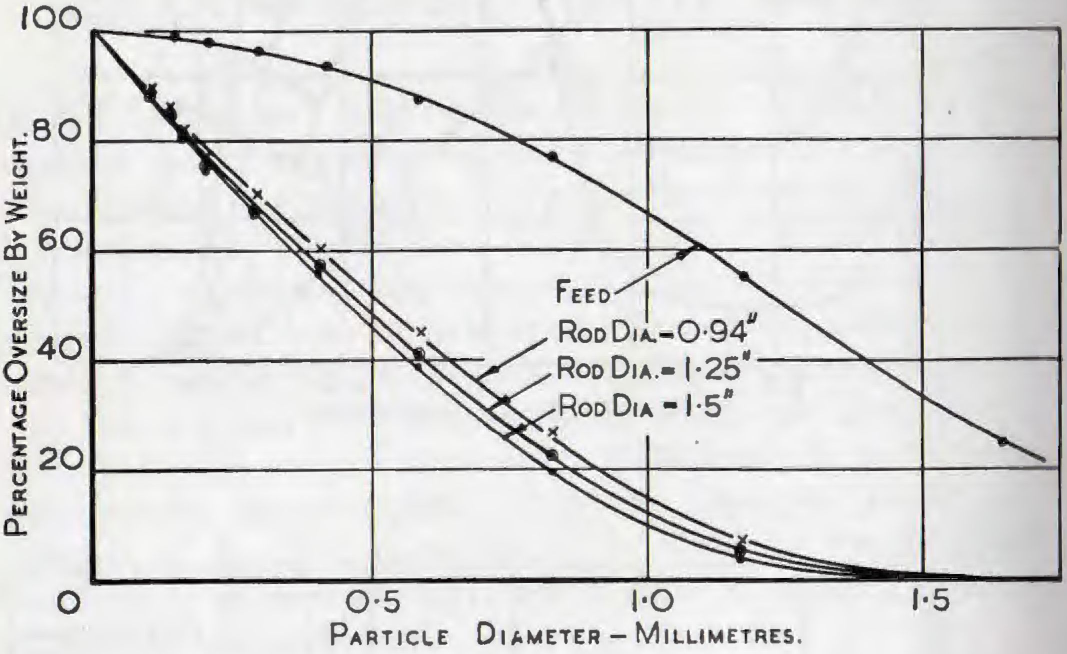

Fig. 6.2

For the rod mill, Fig. 6.2, however, marked selective grinding is not evident; in fact the size distribution curves of the products are very similar for the three sizes of rod tested. This is to be expected since the results are for $N/N_c = 0{\cdot}4$, which would suggest that the charge is cascading and, in such a case, the rods would act as crushing rolls loaded to give a constant force. Thus, the larger particles would be the first to be broken, whatever the size of the rods. It is possible that the grinding characteristics would be different when the charge is cataracting, but evidence on this point is not available.

Examination of Fig. 6.3 shows that variations in the density of the grinding media does not have great effect in *selective* grinding; although the use of the more dense rods does very slightly increase the preferential grinding of the coarser fractions in the feed. Since, however, it is not

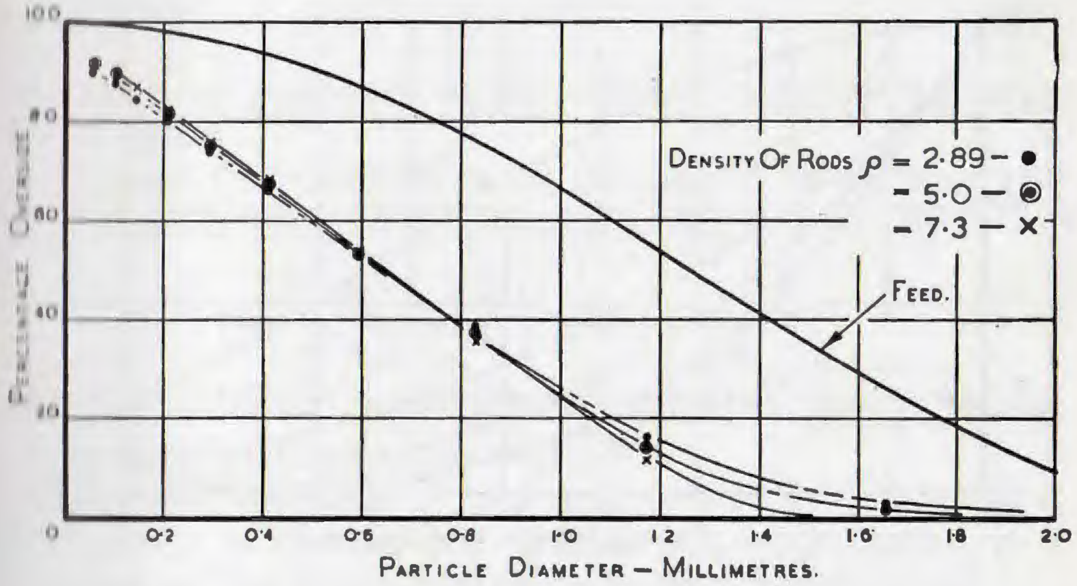

Fig. 6.3

practicable to select grinding bodies made from materials of a wide range of densities, interest in the effects of this variable are theoretical rather than practical. (It should be noted that these tests have been run for times such that equal work has been expended in driving the mill. Thus, it follows that the expenditure of equal amounts of work produce practically identical products from the same feed, regardless of the density of the grinding media.) When the ball charge is fixed, variation of the quantity of ore in the mill brings about no significant selective grinding between the different sizes of material in the feed.

In Fig. 6.4 are plotted curves, again according to Coghill and Devaney, which show the effect of pulp density upon selective grinding and it is at once evident that a more rapid reduction of the larger fractions is obtained with a pulp of moderate thickness than with a very thick one. The probable explanation of this phenomenon is that with a fluid pulp the fine material is washed from between the crushing surfaces, and so is not subject to further grinding. In addition, cleaning the grinding zone of such material leads to a higher efficiency of grinding of the large particles remaining. Thus, both from the point of view of rate of production of surface and of improved grinding of the larger materials, it is desirable that the pulp should be of moderate consistency.

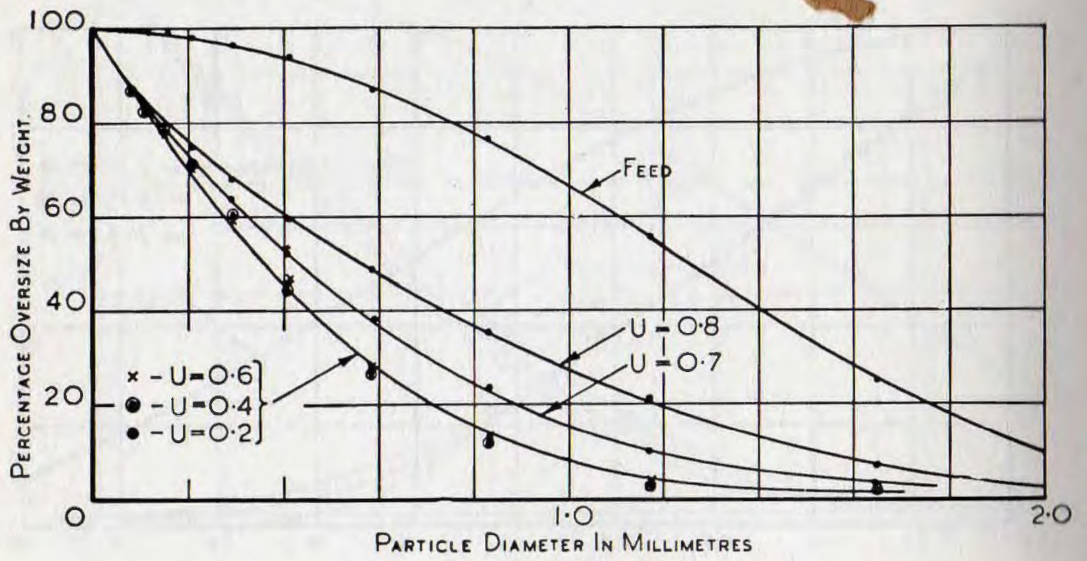

Fig. 6.4

Although there is not much evidence on the point, it appears that the size distribution curves of the products from mills with exit passages of large and small diameter are very similar. Thus selective grinding is but slightly controlled by variation of this dimension.

At this point it is of interest to consider, briefly, the conditions necessary for maximum efficiency in a mill; the efficiency being defined by the equation

$$\eta = \frac{E \times \text{Increase of surface/unit time}}{\text{Energy input/unit time}} \quad .... \quad (6.1)$$

Consideration of the treatment of Chapter 3 will show that, for practical purposes, the power input to a mill is given by

$$P = D^{2.5}\varrho L\left(1 + \frac{0.4\sigma}{\varrho}\right) \cdot {}_P\phi_1'\left(\frac{N}{N_c}\right) \cdot {}_P\phi_3(J) \cdot {}_P\phi_{11}(V) \quad .... \quad (6.2)$$

the effect of variations in the other dimensionless groups being small.

Similarly, consideration of Chapter 5 shows that the rate of production of *specific* surface, $S/t$, is given by

$$\frac{S}{t} = \frac{D^{0.5}\varrho}{E} \cdot {}_S\phi_4\left(\frac{N}{N_c}\right) \cdot {}_S\phi_7(J) \cdot {}_S\phi_9(V) \quad .... \quad (6.3)$$

the variations of the other dimensionless groups being, under normal conditions, negligible.

Here the pre-suffix $P$ or $S$ attached to $\phi$, denotes that the power or the surface production, respectively, is a function of the dimensionless quantity which follows.

The *total* surface, $S_t$, produced is, however, proportional to the product of the specific surface produced and the volume of the mill. Thus the rate of production of *total* surface is given by

$$\frac{S_t}{t} = \frac{D^{2.5}\varrho L}{E} \cdot {}_S\phi_4\left(\frac{N}{N_c}\right) \cdot {}_S\phi_7(J) \cdot {}_S\phi_9(V) \quad .... \quad (6.4)$$

When equations (6.2) and (6.4) are substituted into (6.1) the quantities $D$, $\varrho$, $L$ and $E$ cancel and the mill efficiency is given by

$$\eta = K \cdot \left\{\frac{{}_S\phi_4(N/N_c)}{{}_P\phi_1'(N/N_c)}\right\} \cdot \left\{\frac{{}_S\phi_7(J)}{{}_P\phi_3(J)}\right\} \cdot \left\{\frac{{}_S\phi_9(V)}{{}_P\phi_{11}(V)}\right\} \quad .... \quad (6.5)$$

From this equation it follows that, for given values of $(N/N_c)$, $(J)$, and $(V)$, the efficiency of the mill is independent of size.

In Fig. 6.5 the curves of power input to a dry mill, computed from the results of Chapter 3, and the rate of increase of specific surface, from Fig. 5.17, are plotted on

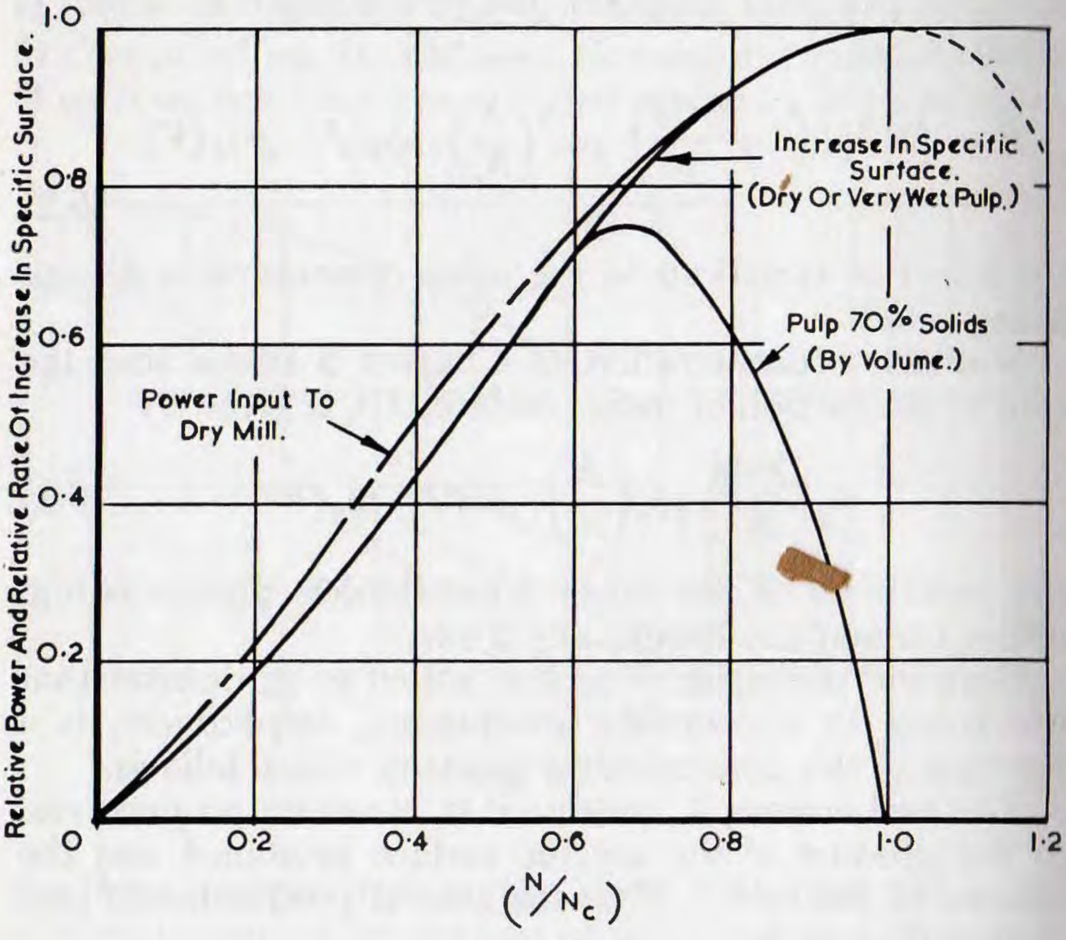

FIG. 6.5

a base of $(N/N_c)$. From this figure it is seen that, for the milling of a dry powder, or a very fluid pulp, there is fair agreement between the curve of power input and that of the rate of production of specific surface, for all speeds up to the critical. For the milling of moderately dry pulps there is also fair agreement for speeds up to about 70% of the critical. Thus, for these two cases the mill efficiency may be considered to be sensibly independent of the speed of the mill.

It is also probable that for speeds above 70% of the critical the power demand would fall away when moderately dry pulps are undergoing milling and so, in this case also, the mill efficiency would remain sensibly independent of the mill speed. Experimental evidence on this point is lacking, however, so a final conclusion cannot be safely drawn, for this case.

It appears to be safe to state, however, that, for all practical milling conditions, the mill efficiency is sensibly independent of the speed of the machine.

The relationship between mill efficiency and mill filling will now be studied.

In Fig. 6.6a is shown the curve relating the rate of production of *specific* surface to the mill filling (see Fig. 5.19) and in Fig. 6.6b is the curve of total surface production, per unit time, deduced from it. The curve Fig. 6.6c is that of power input to the mill and is, in fact, the curve of Fig. 3.5.

The mill efficiency is then deduced by the division of the ordinate of curve Fig. 6.6b by that of Fig. 6.6c and the curve so obtained is plotted in Fig. 6.6d. From this graph it is seen that the mill efficiency is sensibly constant for fillings of up to about 50% and that for greater fillings the efficiency increases rapidly. Thus, it would appear that, for the greatest efficiency, a large filling, say 75%, should be used. This is only partially true, however, for with such a filling the rate of production of specific surface is reduced and so to obtain a product of given fineness the milling time must be increased. If the energy to cause comminution were all that is involved, this would be immaterial, but the energy lost in friction in gearing, bearings, etc., is almost independent of the charge in the mill and this loss is minimized by the reduction of the milling time to a minimum. Furthermore, the reduction of the milling time liberates the plant for other duties and so is economically desirable.

Thus, on balance it appears that lower fillings, with higher rates of specific surface production, are desirable, even though the energy consumption is proportionately larger, and the statement by Gow, Guggenheim, Campbell and Coghill(2) that: "In commercial operation large charges of 55–60% of the mill volume have been found to give lower capacities and efficiencies than loads of 45–50%", is probably well founded.

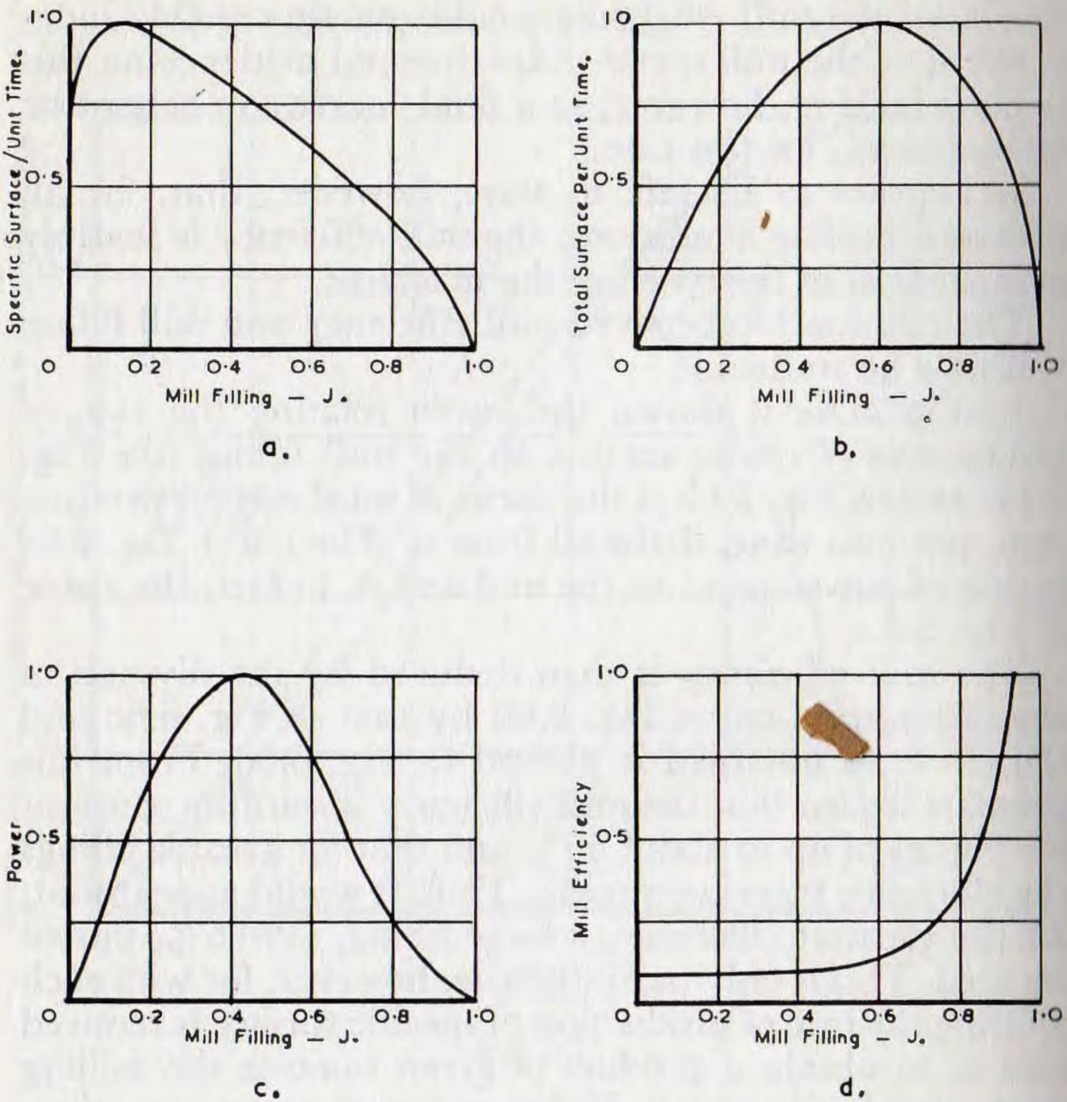

FIG. 6.6

Finally, the relationship between mill efficiency and the quantity of solid material present in the pulp, the parameter ($V$), can be examined.

In this case Fig. 6.7a shows the curve for the rate of production of specific surface, in terms of this parameter (see Fig. 5.21) and in Fig. 6.7b is the curve of total surface deduced from it.

The curve of Fig. 6.7c gives the power demand of the mill, in terms of the variable $V$, so deduced from the treatment of Chapter 3. Although this curve is computed for specific values of mill filling and powder density, it is probable that the form of the curve is applicable to all other practical cases, with an accuracy sufficient for the present purpose.

The mill efficiency is then deduced in the same way as previously and it is seen that the maximum efficiency is reached when the value of $V$ is about 30%.

It must be pointed out, however, that the reduction of the quantity of powder in the mill leads to an increased rate of metal wear. Thus, the quantity of powder cannot be reduced unduly and, on balance, it is probable that the optimum value of the parameter ($V$) is between about 0·6 and 1·0.

Tests by Coghill and Devaney(3) show that the hardness of the balls has some effect upon the rate of grinding in mill. These results are plotted in Fig. 6.8a and it is seen that, both for dolomite and chert, there is an increase in the rate of grinding with increasing hardness of ball. These data require correlating, however, and as a first approximation this can be carried out as follows. In the first place it is clear that the greater rate of grinding with the dolomite is due to the greater grindability, that is the smaller value of $E$, of this material. There is ample evidence however that $E_c$ is 2·75 to 3·0 times $E_d$ so if the ordinates of the curve for dolomite are divided by, say, 2·75 correlation with respect to this variable is introduced. It is also clear, however, that $H$, the relative hardness of the ball to the material undergoing grinding, is an important variable.

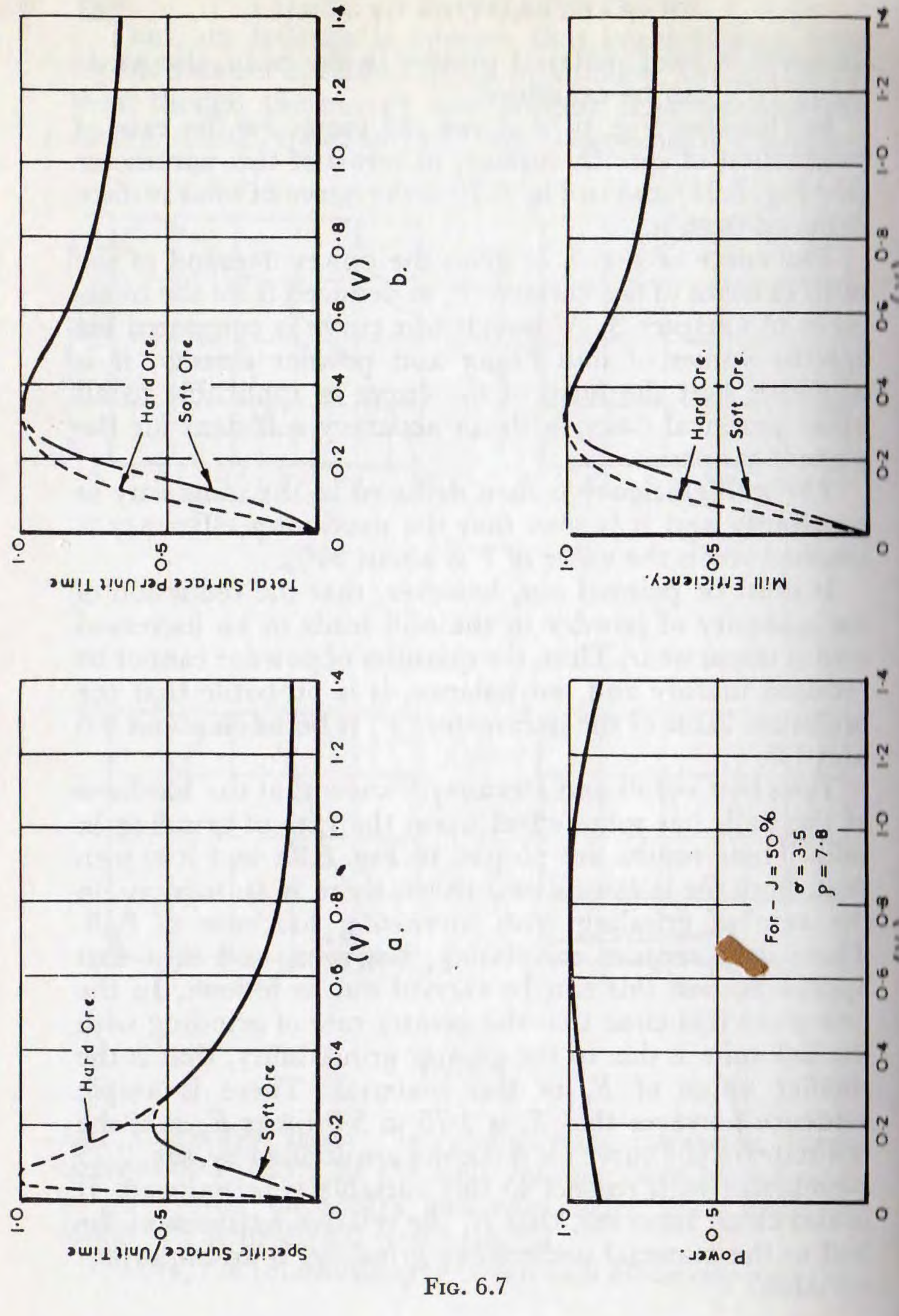
Specific Surface/Unit Time
Power.
Total Surface Per Unit Time
Mill Efficiency.
Hard Ore.
Soft Ore.
Hard Ore.
Soft Ore.
Hard Ore.
Soft Ore.
For – J = 50%
σ = 3·5
ρ = 7·8
(V)
a.
(V)
b.
(V)
(V)
0
0·5
1·0
0
0·2
0·4
0·6
0·8
1·0
1·2
1·4

FIG. 6.7

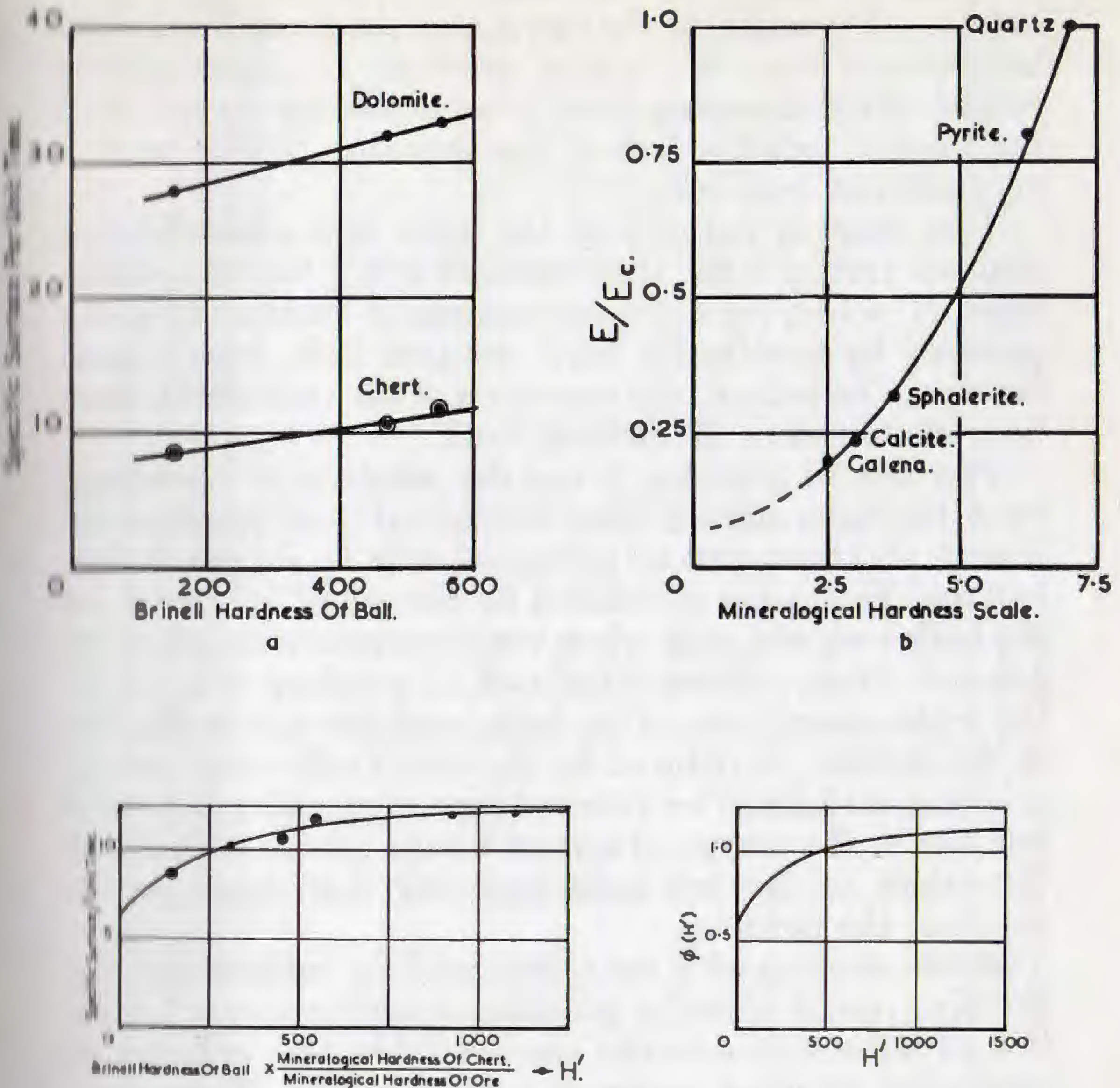

FIG. 6.8

Now a curve by Fahrenwald,(4) shown in Fig. 6.8b, relating grindability to mineralogical hardness, suggests that a reduction in the value of $E$ from 1·0 to 1/2·75 corresponds to a reduction of mineralogical hardness from 1·0 to 0·5 and so the relative hardness of the ball to the dolomite is twice that of the ball to chert. Thus the abscissae values of the curve relating to dolomite must be multiplied by 2·0. When this is done the curve of Fig. 6·8c is obtained and it is seen that there is fair agreement between the results for the two materials. From this it may be deduced that the rate of grinding is approximately halved when changing from grinding the softest ore with the hardest possible ball to the grinding of flint with a very soft cast-iron ball.

This result is reduced to the form of a multiplicative function in Fig. 6.8d. Unit value of $\phi(H')$ has been taken when $H' = 450$, since this corresponds to chert undergoing grinding by moderately hard cast-iron balls, which is, as far as can be judged, the condition of the tests which have been examined in the present work.

The rate of grinding is not the whole story, however, for it has been shown, from theoretical considerations by Rose(5) and from tests on industrial mills by Engels,(6) that ball and liner wear is reduced by the use of *soft* metal for the balls and mill shell when very *hard* materials are to be ground. Thus, although the rate of grinding is reduced, the replacement costs of the balls, and also contamination of the product, is reduced by the use of soft metal for the grinding surfaces. The reduced rate of grinding is doubtless due to the energy absorbed by the plastic and elastic distortions of the soft balls reducing that available for crushing the particles.

Whilst dealing with the question of the influence of the grinding media upon the grinding characteristics of a mill, it is of interest to consider the available data relating to tests upon grinding bodies of various shapes. These data are collected in Table 6.1, and it is seen that, on all counts, spherical grinding bodies give the highest performance figures. Thus, although it might be economical to use large lumps of ore, or "cylpebs" made from scrap drill-rods, when such are available at the plant, there appears to be no economic justification for the use of grinding bodies of shape other than spherical when the bodies have to be purchased at comparable prices.

The increase of power demand brought about by the use of bodies of non-spherical form would be expected;

TABLE 6.1

| Authority | Media | $\frac{S}{t}$ | Power | Surface tons/h.p. hour | Iron loss lb/ton | Merit order* based on: | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Undersize produced | Power consumpt. | Power/lb undersize | Metal wear |
| Coghill and Devaney(7) | Balls | 43·2 | 1·43 | 68·5 | | | | | |
| | Reject Balls | 43·5 | 1·51 | 60·5 | | | | | |
| Fairchild(8) | Balls | | | 1·53 † | 1·74 † | | | | |
| | Cubes | | | 2·15 † | 0·70 | | | | |
| Norris(9) | Spheres | | 144† | | 3·89 | 1 | 1 | 1 | 1 |
| | Cylinders | | | | 4·61 | 4 | 3 | 4 | 4 |
| | Cubes | | 157† | | 4·89 | 5 | 6 | 6 | 5 |
| | Discs | | | | 4·50 | 6 | 4 | 5 | 3 |
| | Modified Spheres | | | | 4·35 | 2 | 2 | 2 | 2 |
| | Modified Cubes | | | | 5·37 | 3 | 5 | 3 | 6 |

* Highest merit—1, lowest merit—5.
† Proportional to these figures.

since the interlocking is tantamount to an increase in the coefficient of friction, which brings about an increased angle of slope of the free surface of the charge, and so an increased displacement of the centre of gravity, which leads to increased power demands.

The size distribution of the balls in the mill charge appears to have no significant effect upon the size distribution of the product, nor upon the rate of grinding, provided that the size distribution of the balls is not too unreasonably chosen. Thus, no significant difference in the grinding characteristics can be detected in the published data on the subject between grinding with balls of uniform size and with balls having a ratio of diameter of largest ball in the charge to the diameter of the smallest ball of about 2 to 1·0 or even greater. This comparative insensitivity of the grinding characteristics to ball-size distribution is fortunate since it means that any convenient method of ball rationing may be adopted.

This at once gives rise to the question of the equilibrium size distribution of the balls in a mill; this distribution being in equilibrium under the conditions of wear which hold during grinding. The problem has been studied by Davis,(10) and, on the assumption that the rate of wear is proportional to the weight of the ball, it has been shown that the size of distribution of the equilibrium charge is given by

$$W = \frac{d_a{}^3 - d_b{}^3}{d_m{}^3 - d_0{}^3} \qquad \ldots . \ (6.6)$$

where $W$ is the percentage by weight of the balls falling in the size interval $d_a$ to $d_b$, $d_m$ is the diameter of the balls fed to the mill and $d_0$ is the diameter at which the balls are rejected.

The results of sieve analysis of the ball charges of large mills, quoted by Davis, are in extremely close agreement with those calculated from equation (6.6) and so the general truth of the theory appears to be established. From this it would appear that a definite equilibrium of the size distribution of the balls exists. So all that can be done in practice is to feed replacement balls of diameter $d_m$ at the correct rate and to remove the balls smaller than diameter $d_0$ either continuously or at fairly frequent intervals. The theory of Davis leads to an expression for the size distribution of the balls in a mill but does not give information as to the rate of loss of metal. This matter will be discussed in a later chapter.

The factors upon which the rate of flow of the pulp through a mill depends appear not to have received extensive investigation. In an article by Anselm,(11) translated by Pearson,(12) a method for the calculation of the time of passage of cement through a mill was given. The basis of the method is that the volume rate of throughput is known and it is assumed that the surface of the powder is coincident with the surface of the ball charge. It follows that the average throughput velocity is the quantity of flow divided by the area open to flow. From this average velocity the time for a particle to traverse the mill may be computed. This treatment does not, however, indicate whether, for a given rate of flow, the surfaces of the powder and the ball charge will coincide or not. For example, with discharge by peripheral ports, it would be expected that the surface of the powder charge would slope quite sharply from the entrance to the exit end. With a trunnion overflow mill, however, by the nature of the design, a steep slope is precluded and there would be but a small "head" to maintain flow.

The small amount of evidence relating to the rate of flow through a mill which is available can again best be studied by the use of dimensional analysis. It would appear to be reasonable to expect that the mass rate of throughput $Q$ tons per hour would depend upon the diameters of the mill, of the ball and the particle, $D$, $d$ and $b$ respectively, upon the length of the mill, $L$, and the speed of rotation, $N$. It would also be expected that the throughput would depend upon the kinematic viscosity of the pulp $\nu$, the density of the pulp $\sigma$, upon the exit diameter of the mill $u$ and upon the mill filling $J$.

When the method of dimensional analysis is applied to these variables it is found that

$$\left(\frac{Q}{D^{5/2}\sigma g^{1/2}}\right) = \phi\left\{\left(\frac{v}{D^{3/2}g^{1/2}}\right), \left(\frac{N}{N_c}\right), \left(\frac{L}{D}\right), \left(\frac{d}{D}\right), \left(\frac{b}{D}\right), \left(\frac{u}{D}\right), (J)\right\} \quad \ldots. \ (6.7)$$

The kinematic viscosity of the pulp would doubtless depend upon the consistency of the pulp, the size of the particles in the pulp and on the cohesive properties of the solid materials when immersed in the liquid phase. Unfortunately there is not nearly sufficient published data to allow a complete study of this equation, but the data of Coghill and Devaney,(13) plotted in Fig. 6.9, indicate that the throughput varies directly as the mill filling, $J$, at least for values of filling up to 50 %. Similarly, from Fig. 6.10, it would appear that the throughput varies directly with the speed of rotation, for speeds up to about 70 % of the critical. At some point above this speed the curve would be expected to become horizontal, since at the critical speed the charge would be spread around the mill and horizontal flow would then be unlikely. Finally, it would be expected that the ratio of the average size of the passages between the grinding media to the particle size would be a controlling factor in the flow of the pulp; since if the particles are larger than the passage dimensions there is mechanical sieving, whilst, if the particles are relatively small, the flow would be limited only by the effective viscosity of the pulp.

Now, from the geometry of a bed of granular materials, the mean pore size is directly proportional to the particle size. Thus, the ratio $(d/b)$ would be a controlling variable. This group is not inconsistent with equation (6.6) since $(d/b) = (d/D) \div (b/D)$.

In Fig. 6.11 are plotted the quantities of flow through a

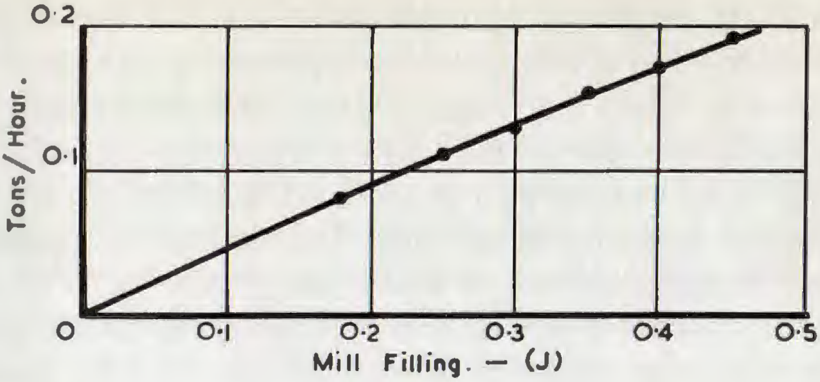

FIG. 6.9

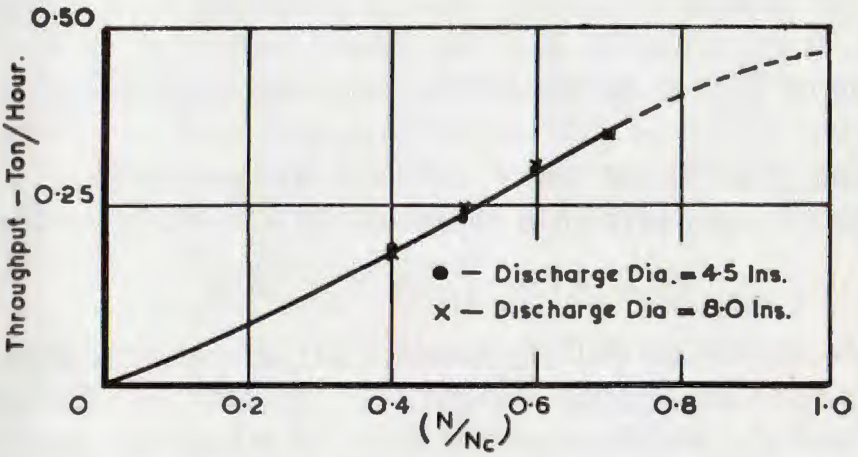

FIG. 6.10

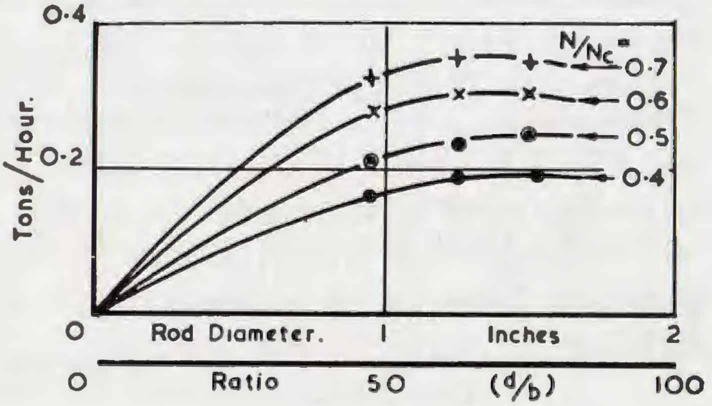

FIG. 6.11

rod mill, in terms of the diameter of the rods. A subsidiary scale also gives the corresponding values of $(d/b)$ and it is seen that the rate of flow is substantially independent of the ratio of $(d/b)$ when the ratio is greater than about 50. This result is in accord with a finding by one of the present authors, Rose and Barnacle,(14) working on the flow of suspensions of solid materials through pipes, that the hydraulic resistance is independent of the particle size provided the ratio of $(D/d)$, where $D$ is the pipe diameter and $d$, the particle diameter, is greater than about 30 to 40.

It is improbable, however, that, in practice, the ratio $(d/b)$ would exceed about 40–50, since this would give rods of excessively large size in relation to the particle. Thus, it is probable that the lower values of $(d/b)$ on the curves of Fig. 6.11 would be relevant and, for this case, $Q \propto d/b$.

Thus, insofar as there are the data available for the analysis, it appears that equation (6.7) can be reduced to

$$Q = KD^{2\cdot5}J.(N/N_c).(d/b). \qquad .... \quad (6.8)$$

From equation (6.7) it is seen that, all other things being equal, the throughput should vary as $D^{2\cdot5}$. Unfortunately data for the verification of this equation are practically non-existent, but those given in Table 6.2 suggest that the observed throughput is roughly in accord with this expression.

TABLE 6.2

| Authority | Mill size Diam. × length (in.) | $J$ | $\frac{N}{N_c}$ | $Q$ observed (tons/hr) | $Q/Q_{19\text{ in.}\times 36\text{ in.}}$ | |
|---|---|---|---|---|---|---|
| | | | | | Calculated | Observed |
| Coghill and Devaney(13) | 19 × 36 | 0·50 | 0·8 | 0·79 | 1·0 | 1·0 |
| | 68 × 48 | 0·50 | 0·8 | 28 | 25 | 35 |
| Norris(9) | 90 × 68 | 0·30 | 0·7 | 15·20* | 26 | 19–25 |

* Estimated figures.

The effect of liner design upon mill performance appears to have received little attention. Clearly, the main function of the liner is to form a removable surface to the mill body, which may be replaced when seriously worn. It is also clear however, that the metal plates which serve this purpose may have a surface which ranges from smooth to one which carries an intricate pattern of raised bars or sunken depressions. The merits of the various types do not appear, however, to have been studied.

As a first step in the analysis of the problem it would be expected that liners would fall into one or the other of two broad classes:

(1) Smooth liners,
(2) "Lifter" liners,

where "smooth" liners are those which have projections insufficient to give appreciable keying between the liner and the ball charge, whilst lifter liners are those which are so heavily ribbed as to give rise to appreciable interlocking between the balls and the liners.

Various common types of liners are illustrated in Fig. 6.12. Although these liners have various patterns of projections, or depressions, to give an amount of interaction between the liner and the grinding medium, it would be expected that wear would round the edges. It is doubtful whether, after some time in service, the performance of a mill with these liners differs appreciably from that of a mill with a smooth surface. Liners furnished with heavy lifter bars are also sometimes used and in such a case the locking of the ball charge to the shell must be very effective. Nevertheless, although a few vague general statements to the effect that a lifter mill gives a product with different size characteristics to that of a smooth mill have appeared, the point does not appear to have been widely investigated. It is probable, however, that, on the grounds of differences in the size characteristics of the products, there exists no sound reason for the use of lifters in preference to the normal smooth liners.

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CHAPTER 7

# SURGING AND VIBRATION IN A MILL

The phenomenon of "surging" in a mill is a subject upon which very little has been written; presumably because it is a condition which cannot be tolerated in mill operation and which must be eliminated by variation of some or many of the physical dimensions or characteristics of the mill or mill charge. The subject is of such importance, however, that it cannot well be omitted from a book dealing with the subject of the tumbling mill. An attempt will therefore be made to give, in this chapter, a review of the present knowledge of the subject.

The phenomenon known as "surging" consists of a pendulum-like oscillation of the whole charge of the mill, within the mill shell. Thus for one part of the cyclical motion the charge is moving around the centre of the mill in the same direction as the mill shell, and during the second part of the motion, in the opposite direction. As a consequence of this oscillatory motion of the charge, the torque necessary to maintain the mill shell in steady motion fluctuates widely and, in certain circumstances, can make such demands upon the driving motor that the circuit breakers are thrown out. This throwing out of the circuit breakers is, however, a lesser evil since the danger is, in such a case, apparent and must be remedied. A more subtle danger is hidden in fluctuations of torque which are insufficient to bring the circuit protective devices into operation. In this case the existence of such fluctuations is not apparent, and the maintenance of fluctuations may lead to the destruction of the driving gearing through fatigue failure. Although, for obvious reasons, but little is made public concerning such failures, it is fairly well known that failure of the gearing of large mills is encountered with a frequency such as to suggest that factors other than mere wear and tear are operative.

The oscillatory motion of the charge, which has been described above, is complicated by the collapse of the leading toe of the charge when the charge comes to rest at the end of the forward motion. This collapse imposes fluctuations of small amplitude but higher frequency upon the fluctuations arising from the main motion; this, in turn, gives rise to a complex motion which involves a great number of harmonics in the frequency spectrum.

Consider now the motion of the charge in some detail and, since, in Chapter 2, it has been shown that the equilibrium surface of the charge approximates to a straight line, the charge may be considered to be a mass of the shape shown in Fig. 7.1. The following mathematical treatment, which has been suggested by Hinsley⁽¹⁾, is then applicable.

The charge in the mill is assumed to be a coherent mass with the centre of gravity at G. The gravitational force $mg$ and the centripetal force $mr\omega^2$ have a resultant $R$ which meets the shell at P. This resultant can, in turn, be resolved into tangential and normal components at P.

Then,

$$\begin{aligned} FR &= m\mu R(r\dot{\theta}^2 \cos \varepsilon + g \sin \psi) \\ &\simeq m\mu R \cos \varepsilon (r\dot{\theta}^2 + g \sin \theta) \qquad \ldots . \ (7.1) \end{aligned}$$

since $\varepsilon$ is small.

Also, from the equations of motion

$$mk^2\ddot{\theta} = mgr \cos \theta - FR$$

or

$$\ddot{\theta} = \frac{gr}{k^2} \cos \theta - \frac{FR}{mk^2} \qquad \ldots . \ (7.2)$$

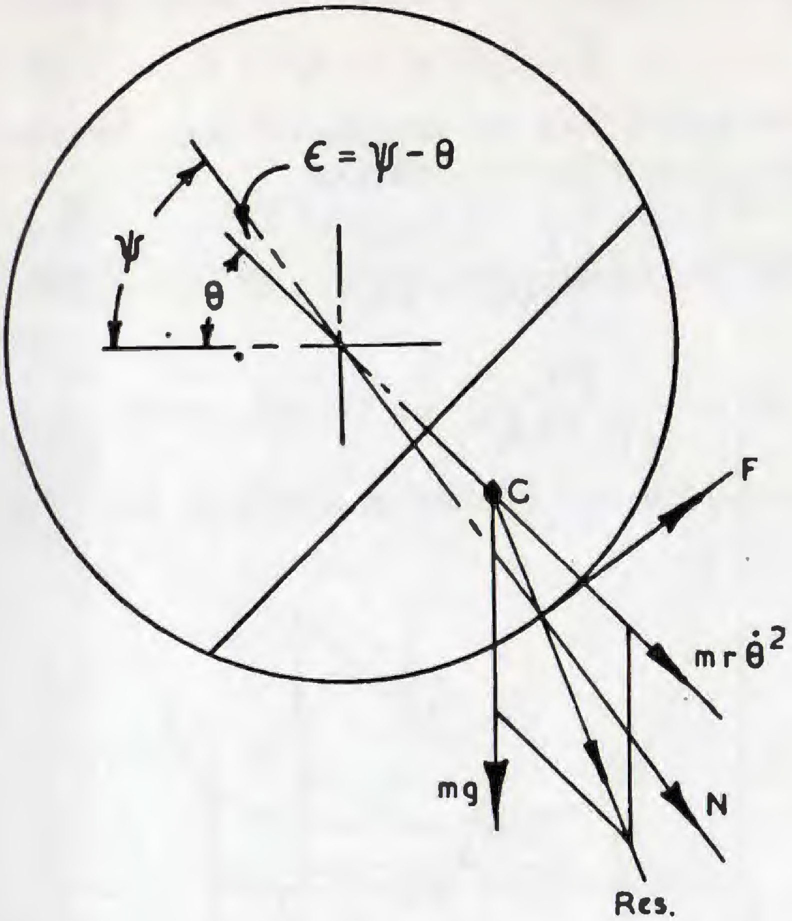

FIG. 7.1

Combining (7.1) and (7.2)

$$\ddot{\theta} = \frac{gr}{k^2}\cos\theta - \frac{\mu Rg}{k^2}\cos\varepsilon\sin\theta - \frac{\mu R}{k^2}r\dot{\theta}^2\cos\varepsilon$$

or

$$\ddot{\theta} + \frac{Rr\mu\cos\varepsilon}{k^2}\dot{\theta}^2 = (\sin\xi\cos\theta - \cos\xi\sin\theta)\frac{g\sqrt{(r^2+\mu^2R^2\cos^2\varepsilon)}}{k^2}$$

which gives

$$\ddot{\theta} + A_1\dot{\theta}^2 = A_2\sin(\xi-\theta) \qquad \ldots\ldots \quad (7.3)$$

where

$$A_1 = \frac{\mu rR\cos\varepsilon}{k^2} \quad \text{and} \quad A_2 = \frac{g\sqrt{(r^2+\mu^2R^2\cos^2\varepsilon)}}{k^2}$$

Let $\phi = \xi - \theta$, so $\dot{\theta} = -\dot{\phi}$ and $\ddot{\theta} = -\ddot{\phi}$, so that

$$\ddot{\phi} - A_1\dot{\phi}^2 = -A_2 \sin \phi \qquad \ldots . \quad (7.4)$$

This equation can be integrated once by use of the integrating factor $2e^{-2A_1\phi}$, whence

$$e^{-2A_1\phi}\dot{\phi}^2 = -2A_2\left\{\frac{-e^{-2A_1\phi}}{\sqrt{(1+4A_1{}^2)}}(\cos\phi + 2A_1 \sin\phi)\right\} + c$$

or

$$\dot{\phi}^2 = \frac{2A_2}{\sqrt{(1+4A_1{}^2)}} \sin(\eta + \phi) + ce^{2A_1\phi} \qquad \ldots . \quad (7.5)$$

where $\eta$ and $2A_1$ are related as shown in Fig. (7.2).

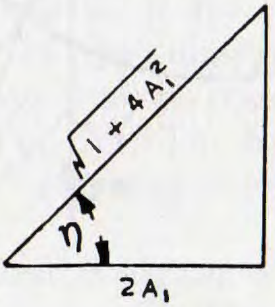

FIG. 7.2

If now the angle $\theta$, at which the charge just slips, is observed by experiment and since, for this condition, $\phi = 0$ the value of $C_1$ the constant of integration, $c$, may be obtained from equation (7.5). Since equation (7.5) is a quadratic there are two solutions, $\phi_1$ and $\phi_2$, to the equation $\dot{\phi}^2 = 0$ which, in practice, are one negative and the other positive.

The expression (7.5) is the equation for the velocity of motion of the charge and the solution can be taken no further by formal methods. For practical purposes, however, it is desirable to know the time for a complete oscillation, since the frequency of the oscillation may then be calculated thus.

The time, $t$, of the descent of the charge is given by

$$t = \int_{\phi_1}^{\phi_2} \frac{d\phi}{\phi}$$

$$= \int_{\phi_1}^{\phi_2} \frac{d\phi}{\sqrt{[\{2A_2/\sqrt{(1+4A_1{}^2)}\} \sin(\eta+\phi) + C_1 e^{2A_1\phi}]}} \quad (7.6)$$

This equation may be solved by graphical and numerical integration.

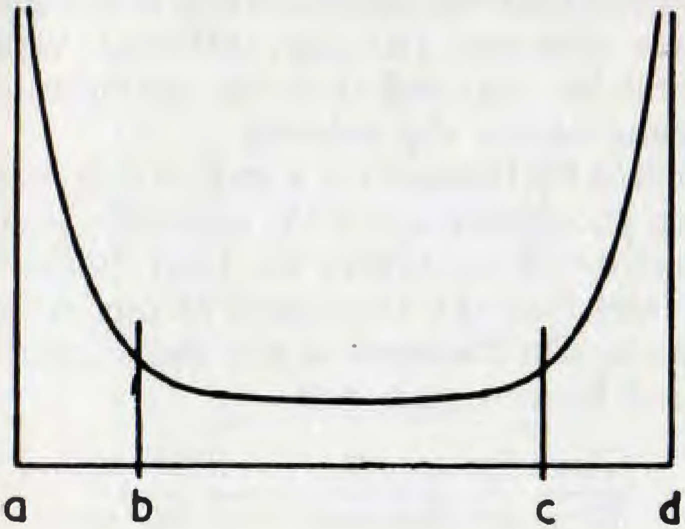

FIG. 7.3

When the function on the right-hand side of equation (7.6) is plotted, a curve of the form shown in Fig. (7.3) is obtained. The portion of the curve between b and c may be integrated by Weddles' rule, or any similar method, but owing to the infinite values, the strips above ab and cd cannot be so treated.

An inverse sine function is therefore fitted to these end strips so that

$$\int_{\alpha}^{\beta} \frac{dx}{\sqrt{(a^2-x^2)}} = \left[\sin^{-1} x/a\right]_{\alpha}^{\beta} \quad \ldots\ldots (7.7)$$

and by the correct choice of $\alpha$, $\beta$, $a$ and $x$ a very close fit may be obtained. Thus the time of *descent* of the load may be computed.

After the load has gripped the shell, the ascent is made at the same speed as the shell. Thus the time of ascent is given by $(\phi_1 - \phi_2)/\omega$; where $\omega$ is the angular velocity of the mill shell.

By the above treatment the frequency of oscillation of the charge may be calculated, provided that certain information, which must be obtained by observation on the mill, is available. Furthermore, the treatment is based on the assumption that the coefficients of static and dynamic friction have constant, although different, values; but in fact it would be expected that the coefficient of friction would depend on the slip velocity.

Experiments by Hinsley on a mill of 6 ft diameter with 40 % filling, show that $\mu_s \simeq 0{\cdot}35$, $\mu_d \simeq 0{\cdot}25$, that there is a lag of about 0·015 sec before the load "locks" on to the mill shell, and that the frequency of oscillation is about 58 per minute. On the basis of the above calculations the following results were obtained.

Time for ascent = 0·233 sec
Time for descent = 0·787 sec
Time for locking* = 0·015 sec

Time for cycle = 1·035 sec

This gives a frequency of $1/1{\cdot}035 \times 60 = 58$ c/min, which is in close agreement with the observed value.

The above treatment is of great interest in that it gives the frequency of surge with considerable accuracy, but it is not of such a nature as to decide, whilst the machine is still on the drawing board, the question of whether a mill of proposed design will surge or not.

In a theoretical study of the problem, one of the present writers, Rose,⁽²⁾ attempted to analyse the motion of the mill charge by use of the mathematical method of topology;

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* Determined by use of an oscillograph.

variations of the coefficient of friction with rubbing velocity being included in this treatment. On the basis of the topological diagram relating to a mill of about 8 ft diameter, together with an assumed but probably reasonable relationship between the coefficient of friction and the rubbing speed, the curve relating the torque to rotate the mill shell at constant speed, on a base of time, was computed and this curve is shown in Fig. 7.4. From this curve it is seen that violent surges in the torque occur; a maximum of about 2·5 times the mean torque being attained at a frequency of about 30 c/min. This ratio of maximum torque to mean torque is in fair accord with an observation by Hinsley, working on the mill mentioned previously, that during surging the current to the motor varied between about 300 and 700 amperes.

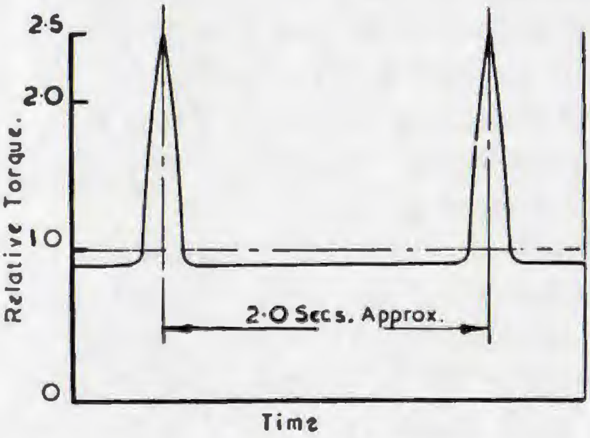

FIG. 7.4

Having regard to the theoretical nature of the treatment of Rose, which involved a number of assumptions as to the values of important parameters, the agreement between the observed and calculated magnitude of the current surge is as close as can be expected. It appears reasonable to conclude that peak values of the torque of at least two or three times the mean value may be encountered during surging.

The topological method is, however, not of a nature such as to be readily used in the design office. In an effort to obtain a simple criterion for the existence, or non-existence, of surging, an experimental investigation was carried out by one of the present authors with a co-worker, Rose and Blunt(3); use being made of the small-scale models and the test apparatus already illustrated in Fig. 3.1.

The basis of this work is that the angle $2\alpha$ through which the centre of gravity of the mill charge oscillates is dependent on the "fluidity" of the charge and this quantity, for reasons already discussed, depends upon the mill diameter, $D$, the ball diameter, $d$, upon the coefficient of friction $\mu$, of the mill charge (if $\mu = 0$ the mill charge would always be in stable equilibrium at the lowest position of the mill) and upon the change of the value of the coefficient of friction with the sliding velocity. Thus if $\mu_v = \mu_0(1 + \phi Bv)$, where $\phi$ denotes some function, the amplitude would depend on $\mu_0$, $B$ and $\phi$.

The amplitude would also be expected to depend upon the speed of rotation of the mill, $N$, and upon the periodic time, $t$, of the oscillation of the mill charge as a pendulum. This would appear to be reasonable since if $N$ and $t$ are such that the mill makes one or $1/n$ revolutions, where $n$ is an integer, during the time that the charge makes one complete cycle, then any roughness of the mill, such as a filling hatch, would produce forces which are synchronized with the oscillation of the charge and the oscillations would build up. If, however, synchronization is not present, then oscillations would be unlikely to build up.

Finally, the amplitude would be expected to depend upon the mill filling, $J$, since both the restoring force acting on the mill charge and the frictional forces, which depend on the number of contact points, depend upon this quantity.

Thus

$$\alpha = \phi(D, d, \mu, t, N, B, J) \qquad \ldots . \quad (7.8)$$

where again $\phi$ denotes "some function of".

If now the methods of dimensional analysis are applied to these variables it is found that

$$\alpha = \phi\{(D/d), (Nt), (\mu), (J), (BDN)\} \quad \ldots . \ (7.9)$$

The analysis may be carried further thus:

Let $N$ be some fraction of the critical speed $N_c$ of the mill so that $N = kN_c$. Now it is easily shown that, for a given mill filling, the periodic time of oscillation of the ball charge varies directly as $\sqrt{D}$ and also that the critical speed $N_c$ varies as $\sqrt{(1/D)}$ and so the value of $(Nt)$ is independent of the size of the mill, thus it follows that, as far as this group is concerned, the results of the model tests are directly applicable to the full-size mill.

Turning attention now to the group $(BDN)$ it follows that, as before $N \propto \sqrt{(1/D)}$ so this group becomes $(B\sqrt{D})$. Thus, it follows that there is some "scale effect", the value of this group increasing as the root of the mill diameter. But on the other hand, the value of $B$ is usually small and so the variations in the value of the group would be expected to be small. Furthermore, the effects rising from the velocity dependence of the coefficient of friction can be, to some extent at least, absorbed by the use of the average between the coefficients of static and dynamic friction. Thus, it would be expected that, in any practical mill, the effect of this group will not be large.

Thus, the basic equation for surging in a mill reduced to, at least for a first approximation,

$$\alpha = \phi\{(D/d), (\mu), (J)\} \quad \ldots . \ (7.10)$$

In accordance with this equation, tests were carried out to establish how the existence, or otherwise, of surging depends on the dimensionless quantities on the right-hand side of the equation.

It has already been shown, in Chapters 2 and 3, that the fluidity of the charge decreases with an increasing number of contact points and with increasing coefficient of friction. Thus, the mill filling, $J$, and the coefficient of friction, $\mu$, are, as far as the fluidity of the charge is concerned, interchangeable, and it would be expected that these quantities would occur in the form of the product $J\mu$. Then, from analysis of the experimental results, it was found that a line could be drawn across the field of a graph, Fig. 7.5, on which $(J\mu)$ is the ordinate and $(D/d)$ the abscissa, and according into which part of the field of the graph the operating points fell, either surging does or does not occur. Thus, in order to establish whether surging is likely to occur or not it is only necessary to establish the product of $J$, the proposed filling of the mill, and $\mu$, the effective coefficient of friction of the ball charge and material undergoing milling, and to plot this value against the proposed value of the ratio $D/d$. If the operating point so obtained is well inside the "non-surging" zone then it is improbable that the mill will surge but if the point is inside the "surging" zone, or dangerously near the boundary line, then surging is likely to occur and the size of the balls or the magnitude of the mill filling, or both,

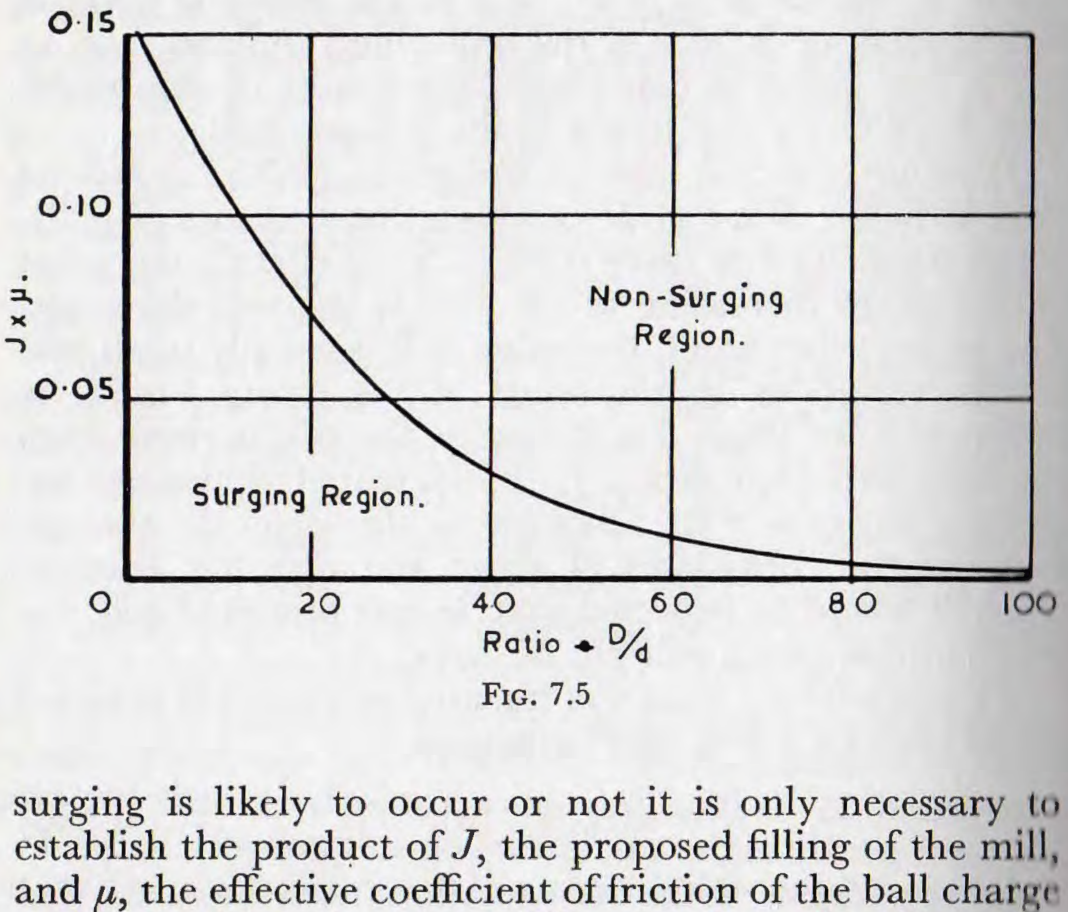

FIG. 7.5

should be varied in order to bring the operating point within the "non-surging" zone.

The foregoing treatment involves a knowledge of the mean of the static and dynamic coefficients of friction of the ball charge and material being processed, and hence of the static and dynamic coefficients individually. Examination of the literature does not reveal values of the coefficients of static and dynamic friction applicable to masses of balls loaded with various types of powder. However, the authors have made rough determinations of these values under conditions which are likely to be applicable to ball mills; that is, balls and powder were placed in an open-bottomed box on a smooth steel surface and the tractive force necessary to move the box measured by means of a spring balance. This method is believed to give results of sufficient accuracy for the present purpose and a number of such results are shown, for purposes of illustration, in Table 7.1.

TABLE 7.1

| Nature of sliding system | Coefficient of static friction $\mu_s$ | Coefficient of dynamic friction $\mu_d$ | Mean value of coefficient of friction $\mu$ |
|---|---|---|---|
| *Steel on steel:* | | | |
| Lubricated with molybdenum disulphide | 0·18 | 0·18 | 0·18 |
| Lubricated with oil | 0·25 | 0·18 | 0·21 |
| Cleaned and dried | 0·37 | 0·30 | 0·33 |
| With free silica powder | 0·42 | 0·41 | 0·41 |
| With free emery powder | 0·44 | 0·42 | 0·43 |

It might be of interest to note in passing that the dependence of surging upon the coefficient of friction, as revealed by model tests, is as shown in Table 7.2, and that

TABLE 7.2

| Conditions in mill | Observation |
|---|---|
| Balls and Mill—Cleaned and dried | Surging occurs |
| Balls and Mill—Oiled | Surging occurs |
| Balls and Mill—Treated with molybdenum disulphide | Surging occurs |
| Balls and Mill—With about 1% of voidage filled with silica powder | No surging |
| Balls and Mill—With about 1% of voidage filled with emery | No surging |

this observation is in accord with experience, since it is unusual, though not impossible, for surging to arise in mills grinding cement clinker, limestone or such similar materials which give rise to a high coefficient of friction. Confirmation of the general accuracy of this criterion is contained in Fig. 7.6, in which the operating points for a large number of mills are plotted; these points being calculated from the data of Taggart(4) and relating to mills between about 4 ft and 10½ ft diameter. Additional data relating to mills between 2 ft and 8 ft diameter grinding molybdenum disulphide and graphite are included; these being supplied by Messrs. Acheson Colloids Ltd.

The data have been reduced on the basis that the coefficient of friction is, for the metalliferous ore, 0·4; a figure which is reasonable for such materials. In fact, the data also refer to mills fitted with various types of liners, "Shiplap", "Wave", etc., but it is probable that all of these liners would approximate to "smooth" after some wear has taken place and so, for safety, these mills should be operated under "non-surging" conditions. Examination of Fig. 7.6 shows that, although some closely approach the boundary, all the points, with one exception, fall within the non-surging zone. Thus, there is fair, though

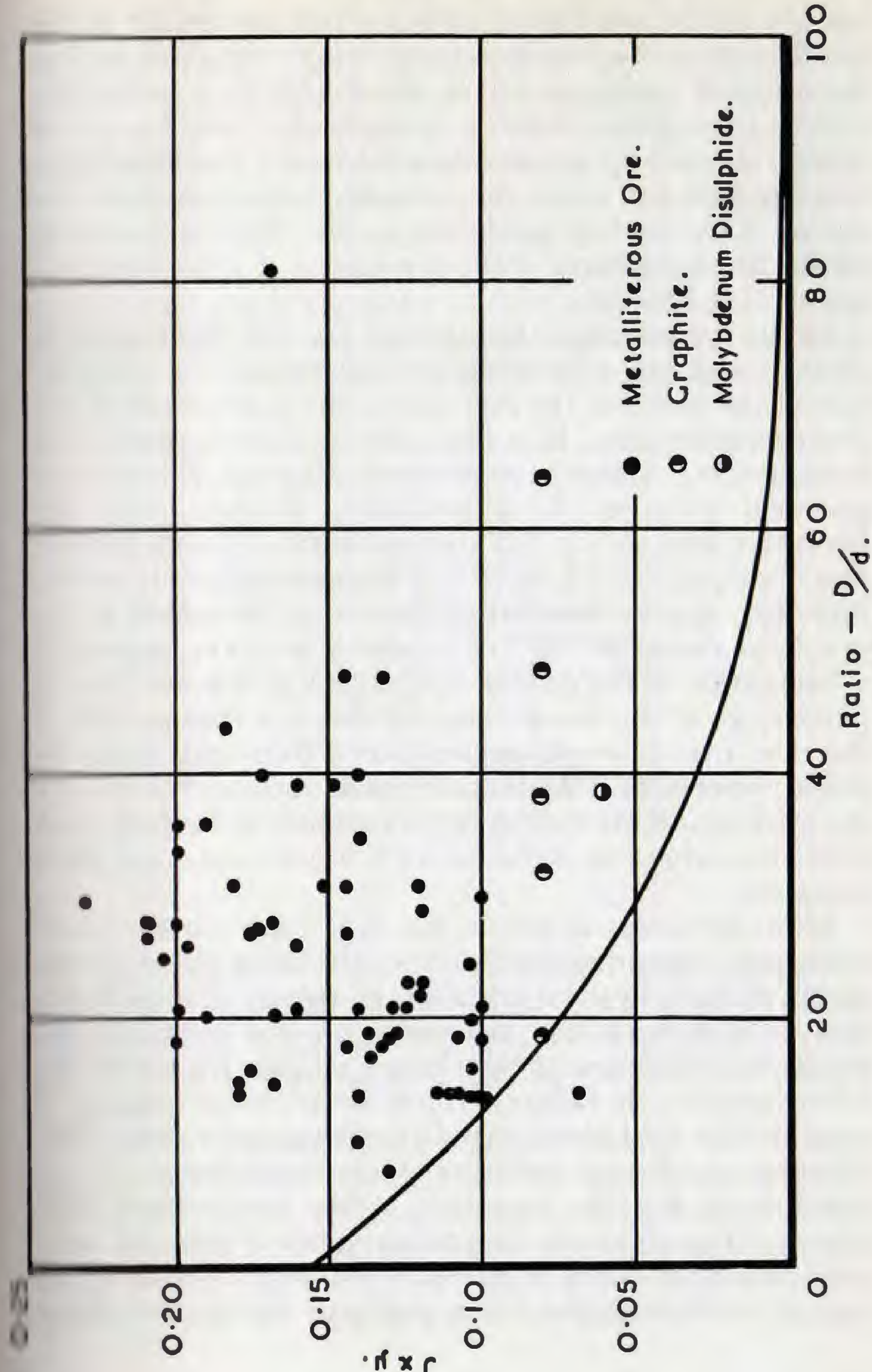
0·25
0·20
0·15
0·10
0·05
0
20
40
60
80
100
J x μ.
Ratio — D/d.
● Metalliferous Ore.
◐ Graphite.
◐ Molybdenum Disulphide.

FIG. 7.6

not absolutely conclusive, evidence that the results of the small-scale tests apply directly to large mills, and so that the effect of variations of the group ($BDN$) is negligible. Strictly, the above criterion is applicable only for values of the ratio ($N/N_c$) greater than 0·15 since it is found that surging does not occur for running speeds less than this value. This further condition is, in practice however, trivial since industrial mills are rarely, if ever, operated at such low speeds.

In the derivation of the curve of Fig. 7.5, the existence, or non-existence, of surging was determined by observation of the sound of the mill and of the fluctuations of the dynamometer arm. It is improbable, therefore, that the surging was completely suppressed; although it was so for practical purposes. It is probable, however, that the boundary line on Fig. 7.5 corresponds to a small fluctuation of torque; say 5 % or 10 % of the mean value. It would, therefore, appear that further lines could be added to the graph, as shown in Fig. 7.7, in which each line represents a fluctuation in the driving torque, equal to some definite percentage of the mean value. If this is a true picture of the case, then it would appear that a given mill might be subject to considerable fluctuations of torque, even though the gross manifestations of the occurrence of surging, such as the throwing-out of the circuit breakers, are not at once apparent.

From the tentative graph, Fig. 7.7, it is clear that small changes of operating conditions could bring about appreciable changes in the fluctuation of torque. It is probable that it is these minor differences in, for example, the coefficient of friction of the charge, which give rise to the often inexplicable failures which occur in the gearing of some mills, whilst identical mills working under apparently identical conditions suffer no such breakdowns. As an example of this the attention of the writers has been drawn, Anon.,(5) to the case of a number of identical tube mills, working under apparently identical conditions, in one of which mills the main gear pair failed after about

10,000 hours' service; whereas the other mills have given completely satisfactory performance over a number of years. The reduction gearing of these mills, which were designed and constructed by makers experienced in the construction of gear trains such as those used in rolling mills and in ship turbine installations, were generously proportioned and apparently adequate for the heaviest duty. If now it is assumed that the fundamental fluctuation of torque is of the order of 30 c/min, which the tests of Hinsley and the calculations of Rose indicate as being probably reasonable, and that the reduction ratio of the slow speed pair is 5 to 1, then 10,000 hours of service would correspond to about 3,000,000 cycles of stress on the gearing. It is significant that it is at this order of numbers of stress cycles that fatigue failure would be expected to occur.

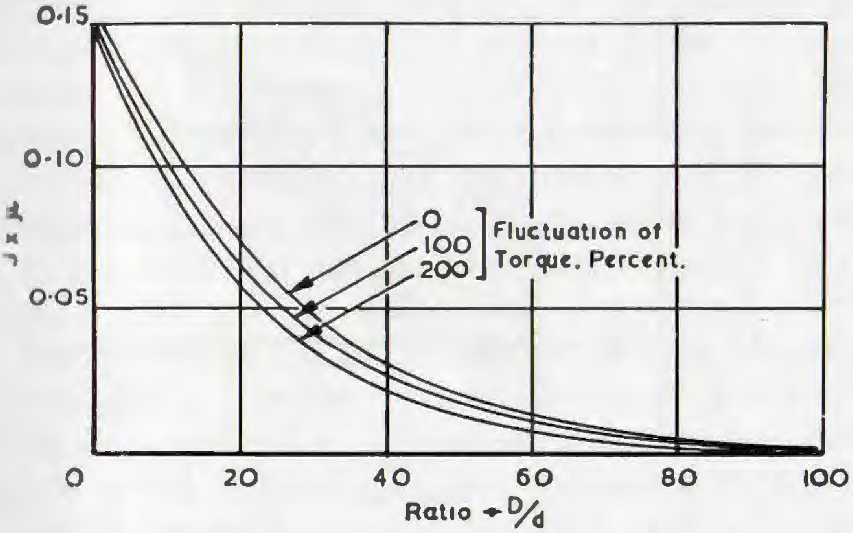

FIG. 7.7

If, in the absence of an exact knowledge of the composition of the gear material and the heat treatment employed, it is assumed that the fatigue characteristics of the gear material are approximately the same as those for which the diagram Fig. 7.8, quoted by Morley,(6) was derived, then this diagram may be used for the further analysis of the present problem. The diagram of Fig. 7.8 is such that, for an indefinite number of reversals before failure, the working stresses must fall within the shaded area.

Now suppose that the minimum stress, corresponding to the mean driving torque, is 20 tons/in$^2$. It follows that, if failure is not to occur within a limited number of applications of the stress, the maximum stress must not exceed about 25 tons/in$^2$; that is, the allowable fluctuation of torque must not exceed about 25 % of the mean value. However, it has already been suggested by Fig. 7.7, that

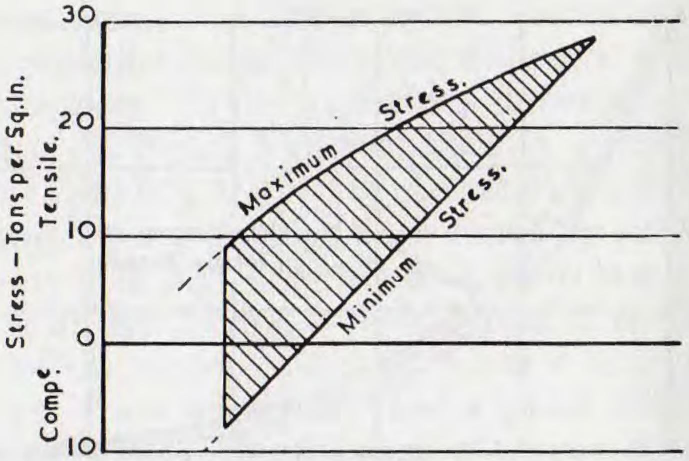

FIG. 7.8

fluctuations of this magnitude may well arise from quite moderate changes in mill conditions, possibly by, say, 10 % variation in the value of the coefficient of friction of the charge, and that such changes could well be brought about by differences in pulp density, particle size, surface finish of the balls and mill, etc., which are so small as to be normally regarded as unimportant. Thus, it would appear that the unexplained gear failures encountered in practice might arise from differences in the physical conditions of the mill liner, ball charge and powder characteristics. These are so small as to be regarded as unimportant but do, in fact, have a profound effect on the torque fluctuations transmitted to the gearing. The problem is still further complicated by the fact that the stresses involved will depend upon such factors as the rigidity of the drive, the torque-speed characteristics of the motor, etc. Thus, for example, if the motor characteristics are such that the mill is allowed to lose speed as the torque increases, the load on the gearing will be lower than if the motor maintains the speed of rotation more or less constant. For these reasons exact calculations as to the torque fluctuations are impracticable, but there remains no reasonable doubt that a mill should be operated as far within the "non-surging" zone of Fig. 7.5 as is possible.

It has already been pointed out that, owing to the periodicity of any surging motion of the charge within the shell and to the collapse of the toe of the charge, the fluctuations of the torque transmitted by the driving mechanism will contain a very large number of harmonics. It is, therefore, possible that resonance might occur at some frequency, between a mill and nearby apparatus which, if the adjacent equipment were sensitive to vibration, could give rise to difficulties, the origin of which might not be easily traced. Problems of this nature have been discussed in a recent article, Anon.,(7) in which it is shown that appreciable vibration may be transmitted through a piled foundation; the vibration being transmitted down the piles beneath the mill, through the bedrock and then emerging through other piles beneath the neighbouring equipment. This effect would not be expected to be so marked with friction piles, since there would then be no underlying rock to transmit the vibrations and the clay itself would introduce considerable damping. Nevertheless, it would appear that the correct design of foundations from the viewpoint of the transmission of vibration, is essential.

In the same article, it is mentioned that some vibration emanated from the mill gearing and it was suggested that this condition could be alleviated by the use of fibre or spiral toothed driving pinions in the gear train. In view of the discussion in the earlier part of this chapter, however, it appears that attention to the conditions within the mill shell might be more effective than such modifications to the gear train itself. A further type of vibration which might well be mentioned is the vibration of the mill shell, as a beam, under the influence of the falling charge. Probably the greater part of this vibration is absorbed by damping in the metal of the shell, the joints between the liner plates, etc., and, in a ball or tube mill there appears to be no need to introduce any special measures for its suppression. In a rod mill, however, the individual rods give a more violent impact when they make contact with the liner or the toe of the charge. It appears to be common practice to insert billets of wood between the liners and shell of such mills in order to attenuate the shock and to introduce some damping.

The particularly destructive action of heavy rods falling on the mill liners emphasizes, however, the importance of running rod mills at a speed such that the rods impinge on the toe of the charge and not upon the liners.

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## CHAPTER 8

# METAL WEAR IN MILLS

THE cost of replacement of the metal worn from the working surfaces of the balls and liners of a mill often represents a considerable fraction of the cost of operation of the plant and so is of some real economical importance. In spite of this importance, however, the amount of information available on this subject is amazingly small and, unfortunately, is insufficient to form the basis of more than a very tentative analysis of the problem. As far as is known the only attempt to derive a theory for metal wear is that of one of the present authors, Rose,(1) and the relevant parts of this treatment will be outlined briefly here.

The basic idea underlying this treatment is that particles of the material undergoing milling will, upon contact, be driven into the surface of the balls at a rate which depends upon the relative hardnesses of the ball and particle, the quantity of powder present and upon other such factors. It is also assumed that, provided there is relative tangential motion between a pair of balls, one or other of the embedded particles will, upon making contact, as in Fig. 8.1a, be torn out. This then leads to the conclusion that the particles will be torn from the metal surfaces at a rate which is a function of the relative hardness of the ball and particle, of the number of particles embedded, the relative sliding velocity, and similar factors.

When the rates of embedding and removal of particles are equal there will be an equilibrium number of particles embedded in the metal surfaces or, in other words, a certain portion of the surface of each ball will be "armoured" with embedded particles. If now, an embedded particle in one ball traverses the "unarmoured" part of the surface of a neighbouring one, as in Fig. 8.1b, then metal wear will take place. In the work cited the calculation of such wear has, to a first approximation, been carried out and it is suggested that the relationship between rate of wear and relative hardness of ball and particle has the form shown in Fig. 8.2, whilst the relationship between rate of wear and quantity of powder in the mill has the form of Fig. 8.3.

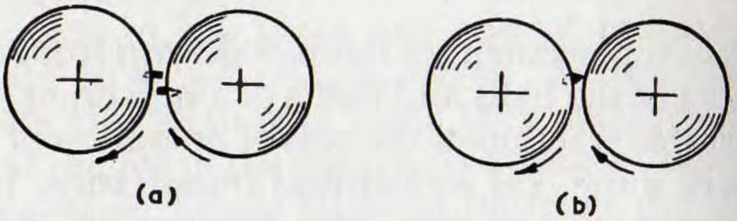

FIG. 8.1

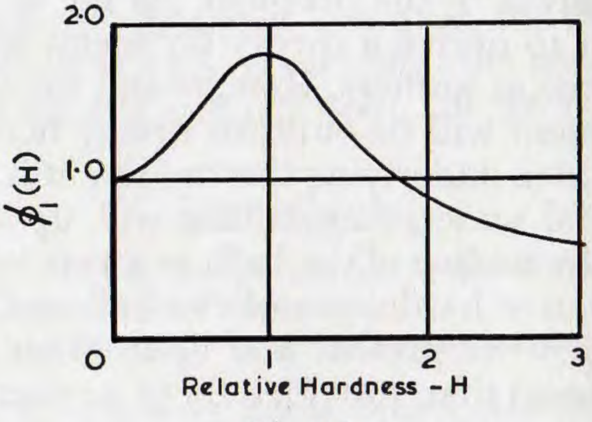

FIG. 8.2

Although the derivation of these curves is completely theoretical there is some qualitative evidence in support of their validity. Thus for example, it is clear from Fig. 8.2, that if a material is being ground by balls of approximately equal hardness ($H = 1$), then the ball wear may be reduced either by increasing or by decreasing the value of $H$. If, however, a hard mineral, say of Brinell Number about 500 to 600, is being milled, it is not practicable to use balls of appreciably greater hardness and so the reduction of wear by increasing the hardness of the grinding surface is not possible. In such a case, however, the rate of wear can be reduced by the use of very soft metal (by reducing *H*) and so carrying the operating point well to the left of the maximum of curve Fig. 8.2. This improvement is, of course, brought about by the increased "armouring" of the surface of the balls by the particles. Conversely when a very soft material is being ground, the ratio *H* will be greater than unity, even with soft metals, and so the operating point is on the right-hand side of the peak. In this case reduced wear is obtained by *increasing* the hardness of the metal.

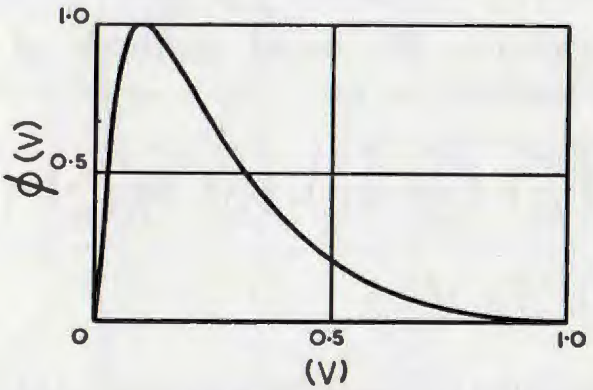

Fig. 8.3

These conclusions are confirmed by the practical observations of Engels⁽²⁾ that for the grinding of soft materials a *hard* mill body and balls should be used, but for grinding hard materials a soft but tough metal should be used for the grinding surfaces. Similarly, the curve of Fig. 8.3, is confirmed by Smith,⁽³⁾ who states that it is well known to mill operators that an improved fineness, in a given time, is attained when the quantity of powder in the mill is reduced, but that this improvement is obtained at the expense of increased iron wear.

In order to analyse the various published data, advantage may well be taken of the simplification offered by the method of dimensional analysis and this will be used as a basis of the subsequent treatment.

It would be expected that the total wear of metal, $W_t$ lb, would depend upon the relative hardness of ball and particle, $H$, upon the size of mill, ball and particle, $D$, $d$ and $b$ respectively, upon the length of the mill, $L$, upon the speed of the mill, $N$, and upon the density of the ball material, $\varrho$. It would also be expected to depend upon $E_m$, the energy necessary to increase the surface of the metal by unit amount, upon the mill filling, $J$, upon the fractional volume of the powder, $V$, upon the pulp consistency, $U$, upon gravity, $g$, and upon the time of milling, $t$.

This may be written symbolically as

$$W_t = \phi\{D, d, b, L, H, E_m, \varrho, N, g, t, J, V, U\} \quad \ldots . \quad (8.1)$$

and, after applying the usual methods of dimensional analysis, this leads to

$$\left(\frac{W_t}{D^3\varrho}\right) = \phi\left\{\left(\frac{gt^2}{D}\right), \left(\frac{d}{D}\right), \left(\frac{b}{D}\right), \left(\frac{L}{D}\right), \left(\frac{E_m}{D^2\varrho g}\right), \left(\frac{DN^2}{g}\right), (H), (J), (V), (U)\right\} \quad \ldots . \quad (8.2)$$

This can be simplified, however: since $(DN^2/g)$ is, from equation (3.7), equivalent to $(N/N_c)^2$.

$$\left(\frac{W_t}{D^3\varrho}\right) = \phi_1(H) \cdot \phi_2\left(\frac{E_m}{D^2\varrho g}\right) \cdot \phi_3\left(\frac{N}{N_c}\right) \cdot \phi_4\left(\frac{d}{D}\right) \cdot \phi_5\left(\frac{b}{D}\right) \times \phi_6\left(\frac{L}{D}\right) \cdot \phi_7(J) \cdot \phi_8(V) \cdot \phi_9(U) \quad \ldots . \quad (8.3)$$

First examine the functional relationship $\phi_1(N/N_c)$. Data by Coghill and Devaney(3) and Gross,(5) relating to metal wear, are plotted in Fig. 8.4, and on the same graph the curve relating the rate of production of specific surface to the group $(N/N_c)$, taken from Fig. 5.17, is shown for comparison. It is at once seen that there is close agreement between the rate of production of specific surface in a mill and the rate of ball wear, insofar as these quantities depend on the speed of rotation of the mill. It is also found that for slow speeds, $(N/N_c) < 0{\cdot}6$, the functional relationship can be replaced with reasonable accuracy by $\phi_1(N/N_c) = K(N/N_c)^{1{\cdot}30}$ and for the higher speeds, $(N/N_c) > 0{\cdot}6$ by

$$\phi_1(N/N_c) = K(N/N_c)^{0{\cdot}8}.$$

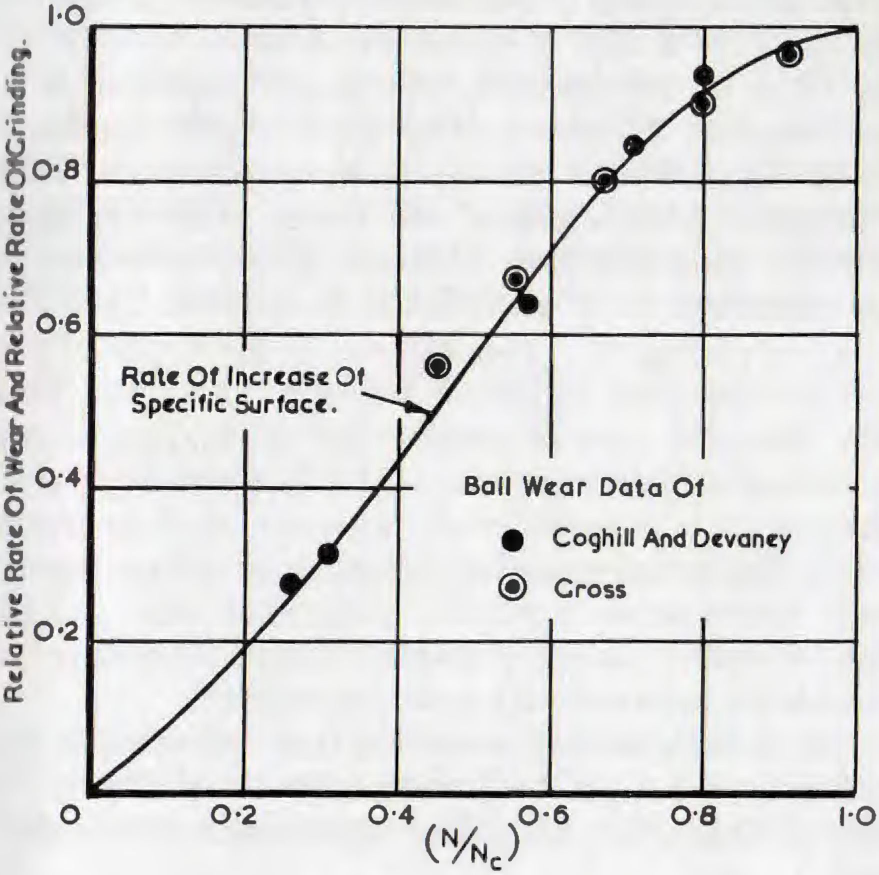

FIG. 8.4

The decrease in the rate of ball wear at higher speeds would be expected since, whilst decreasing the tangential motion, the cataracting would increase the frequency and intensity of impacts which would promote armouring. As the centrifuging condition is approached the number of balls in action decreases, which again would reduce ball wear. Thus the curve of Fig. 8.4 appears to be of reasonable form.

The close agreement between the points and of the curve of Fig. 8.4 suggests that the metal wear per unit increase in surface attained should be approximately independent of the speed of the mill. A study of Chapter 3 also shows that, for a mill of given size and for speeds of up to about 70% of the critical, the power input to a mill is proportional to the speed of rotation. Again insofar as the curve of Fig. 8.4 may be replaced by a straight line over this range, it also leads to the fairly generally accepted conclusion that the rate of wear is proportional to the power absorbed in a mill. Thus it appears that the ball wear is, very roughly, proportional to both rate of production of surface and to power input to the mill. This also implies that the rate of production of surface is roughly proportional to power input; which has also been deduced in Chapter 6. It is considered, however, that to attempt to calculate the metal wear by reference to power input or to surface increase is artificial and that the calculation should be treated as a separate problem involving its own relationships between the mill parameters.

The next relationship which is fairly obvious is $\phi_6(L/d)$ since the total rate of ball wear must vary directly as the length of the mill; all other quantities remaining unaltered.

Turning now to the group $(E_m/D^2\varrho g)$ it would appear that, on logical grounds, the relationship must take the form

$$(W_t/D^3\varrho) = K(D^2\varrho g/E_m) \qquad \ldots. \quad (8.4)$$

since, all other things being equal, the total wear, $W_t$, must be inversely proportional to the value of $E_m$. Unfortunately there is no evidence against which this relationship can be tested directly.

Turning next to the function $\phi_5(b/D)$ there are limited relevant data by Mortsell$^{(6)}$ which suggest the general form of this relationship. These data are plotted in Fig.

8.5 and it is seen that the curve has a sharply defined maximum. The probable explanation of the form of this curve is that the "armouring" becomes less effective but the abrasion increases with increasing particle size until the maximum is reached. After this, even though the "armouring" is still ineffective, the particles separate the metal surfaces, acting rather like ball bearings, and so, with increasing size, reduce the abrasive wear of the metal. It should be noticed that, for the right-hand branch of the curve, which is the practical range, wear varies approximately inversely as $(b/D)$ or, thus, directly as $(D/b)$, and so with the mill diameter. Conversely, for the left-hand portion of the curve the wear varies almost exactly as $(b/D)$; or thus as $(D/b)^{-1\cdot 0}$. The left-hand branch is clearly for large balls grinding a fine powder and the right-hand side is for small balls with coarse material present.

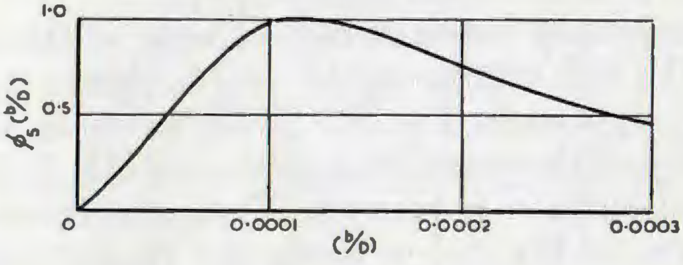

FIG. 8.5

Considering next the effect of relative hardness on the rate of ball wear; a theoretical relationship has been presented in Fig. 8.2. Data for the checking of this relationship are almost completely lacking, but a few values of ball wear, for different Brinell hardnesses of ball, published by McLeod(7) are plotted in Fig. 8.6. This curve shows the general form of Fig. 8.2, but definite correlation cannot be made since very mixed minerals were milled and the corresponding mineral hardness is unknown. Thus, in the absence of more complete data it must be assumed that the curve of Fig. 8.2 represents this relationship.

The influence of the ratio of mill size on the rate of wear $\phi_4(d/D)$ can more conveniently be written in the form $\phi_4(D/d)$, and in Fig. 8.7 are plotted the data of Coghill and Devaney⁽⁴⁾ and Gross,⁽⁵⁾ relating to the wear of balls, when the charge consists of balls of uniform size, and of McLeod⁽⁷⁾, relating to the wear of balls of size $d$ in a charge consisting of balls of mixed sizes. A simple change of the vertical scale of these curves brings them into coincidence and so they may be generalized into the curve of Fig. 8.8; this curve representing the function $\phi_4(D/d)$ insofar as it may be derived from these data. It is also seen from this figure that, over the practical range of $10 < (D/d) < 50$, the function may be replaced by $\phi_4(D/d) = 0{\cdot}556(D/d)^{0{\cdot}16}$, with an error of less than about 10 %

The relationship between rate of wear of the metal surfaces and the ball charge in the mill is shown by the data of Gross,⁽⁵⁾ plotted in Fig. 8.9. This curve is seen to be, very roughly, of the same form as that of Fig. 5.18, relating to the total surface produced. The agreement with the general form of Fig. 3.5 relating the power input to the mill, is not so good for the higher mill fillings; the rate of metal wear being much higher than the power input. This observation is in accordance with the hypothesis of "armouring" since, with high mill fillings the balls are constrained to roll, with considerable abrasion, but impact, which leads to effective armouring, is suppressed.

The only data relating to the effect of the quantity and consistency of the pulp on the ball wear appear to be those of Gross.⁽⁵⁾ Unfortunately, these are insufficient to form the basis of a complete analysis. Furthermore these data appear to be defective in that they are based on runs of 21 minutes' duration and it appears that this period is insufficient to allow equilibrium to be reached; that is for the armouring to have become established. However, in the absence of more complete data these must be accepted tentatively. It appears that, as a first approximation, the effects of both pulp quantity and pulp density can be stated in terms of the volume of solid material present by

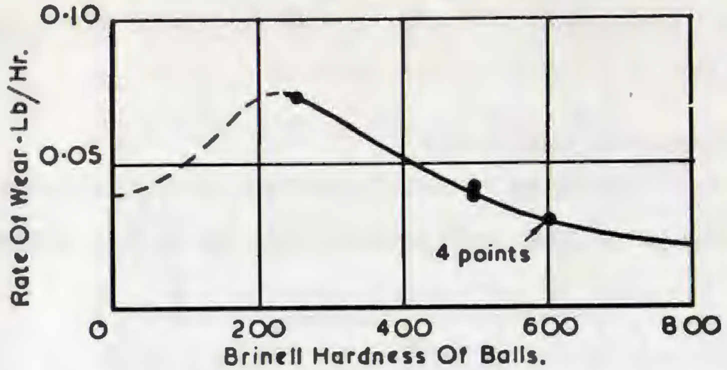

FIG. 8.6

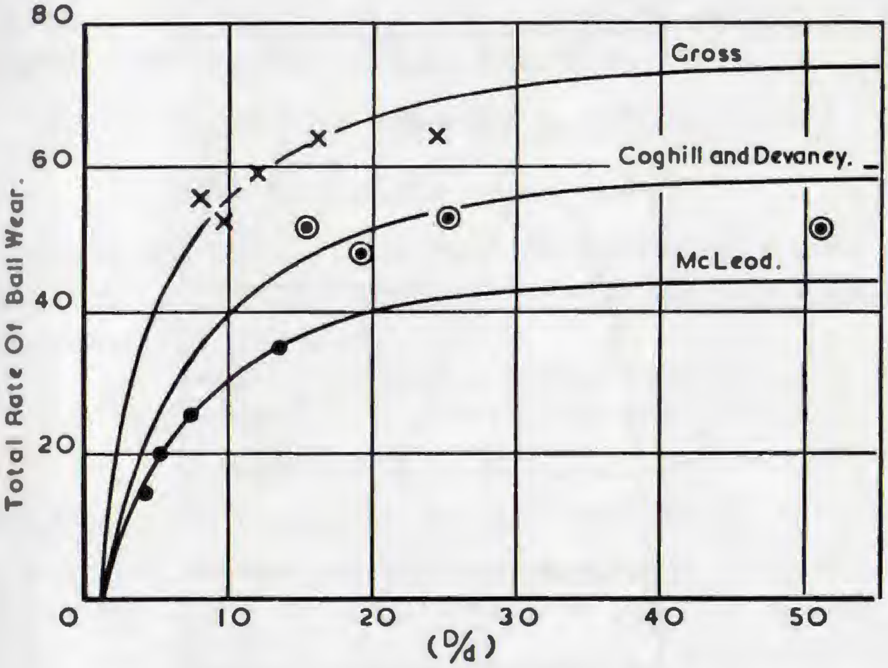

FIG. 8.7

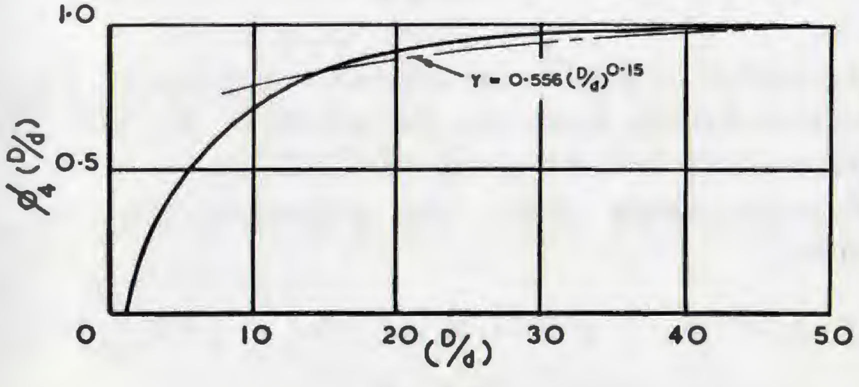

FIG. 8.8

use of the function $(V)$ where, as before, the quantity $V$ is defined by

$$V = \frac{\text{Volume of particles} + \text{Volume of space between the particles}}{\text{Volume of space between the balls (or rods)}} \quad (8.5)$$

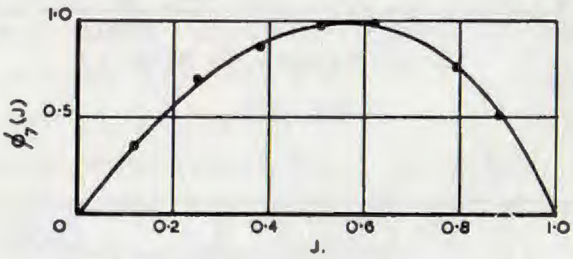

FIG. 8.9

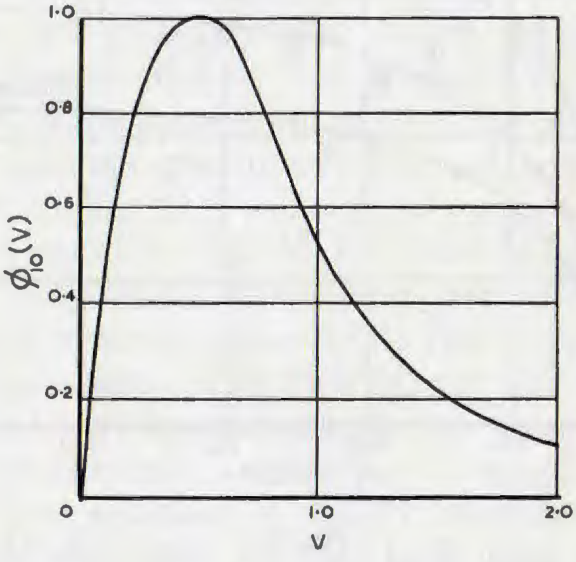

FIG. 8.10

and the curve of $\phi_{10}(V)$ so deduced is given in Fig. 8–10. This curve should, however, be taken as only the roughest approximation to the actual relationship.

Collecting these data, the equation for ball wear becomes

$$W_t = KD^{3\cdot5}E_m^{-1.0}\varrho^{1/2}Lt.\phi_1(H).\phi_3\left(\frac{N}{N_c}\right).\phi_4\left(\frac{D}{d}\right).\phi_5\left(\frac{b}{D}\right) \times \phi_7(J).\phi_{10}(V) \quad \ldots\ldots (8.6)$$

where $\phi_1(H)$ is read from Fig. 8.2,
$\phi_3(N/N_c)$ is read from Fig. 8.4,
$\phi_4(D/d)$ is read from Fig. 8.8,
$\phi_5(b/D)$ is read from Fig. 8.5,
$\phi_7(J)$ is read from Fig. 8.9,
and $\phi_{10}(V)$ is read from Fig. 8.10.

If now $E_m$ and $\varrho$ are regarded as sensibly constant for any steel used in the manufacture of balls and liners, and if $\phi_4(D/d)$ is replaced by $(D/d)^{0\cdot 15}$ and $\phi_5(b/D)$ by $(D/b)$, which may be done over the practical range of these variables, then equation (8.6) becomes

$$W_t = K_1 D^{4\cdot 65} d^{-0\cdot 15} b^{-1\cdot 0} Lt \,.\, \phi_1(H) \,.\, \phi_3(N/N_c) \times \phi_7(J) \,.\, \phi_{10}(V) \quad \ldots . \quad (8.7)$$

In practical milling the ball diameter is usually approximately a constant multiple of the particle diameter, so this equation becomes

$$W_t = K_2 D^{4\cdot 65} d^{-1\cdot 15} Lt \,.\, \phi_1(H) \,.\, \phi_3(N/N_c) \times \phi_7(J) \,.\, \phi_{10}(V) \quad \ldots . \quad (8.8)$$

If the relative hardness, $H$, the mill filling, $J$, and the ratio of solids, $V$, are maintained constant, then, from the relationships $\phi_1(N/N_c) = K(N/N_c)^{1\cdot 30}$ for low speeds and $\phi_1(N/N_c) = K(N/N_c)^{0\cdot 8}$ for high speeds, this relationship becomes:

For low speed,

$$W_t = K_3 D^{4\cdot 65} (N/N_c)^{1\cdot 3} d^{-1\cdot 15} Lt \quad \ldots . \quad (8.9)$$

and for high speeds,

$$W_t = K_4 D^{4\cdot 65} (N/N_c)^{0\cdot 8} d^{-1\cdot 15} Lt \quad \ldots . \quad (8.9a)$$

The number of balls in the mill $\propto D^2 L/d^3$ so the wear per ball $W_b$ is given by

For low speed,

$$W_b = K_5 D^{-2\cdot 65} d^{1\cdot 85} (N/N_c)^{1\cdot 3} t \quad \ldots . \quad (8.10)$$

and for high speed,

$$W_b = K_6 D^{2\cdot65} d^{1\cdot85} (N/N_c)^{0\cdot8} t \quad \ldots . \quad (8.10a)$$

These equations should be contrasted with those of Hukki(8):

For low speed,

$$W_t = KD^{2\cdot5}(N/N_c)d^{-1\cdot0}Lt \quad \ldots . \quad (8.11)$$

and for high speed,

$$W_t = KD^{2\cdot5}(N/N_c)Lt \quad \ldots . \quad (8.11a)$$

which lead to:

For low speed

$$W_b = KD^{0\cdot5}(N/N_c)d^2 t \quad \ldots . \quad (8.12)$$

and for high speed,

$$W_b = KD^{0\cdot5}(N/N_c)d^3 t \quad \ldots . \quad (8.12a)$$

For low speeds the present equation is in fair agreement with that of Hukki insofar as ball diameter is concerned, but at high speed, and in all other respects the agreement is not so good.

The formulae of Hukki are, however, derived from first principles on the basis of clearly defined assumptions. These assumptions are that, for slow speed, the rate of wear is proportional to the rubbing velocity and, for high speed, the wear is proportional to the force of the blow. The first assumption appears to be quite reasonable but the second is not so free from objection. In the first place, the question of the comminution of a material by a blow is involved, and this has already been shown to be a complicated question. Even, however, if the damage is assumed to be proportional to the intensity of a single blow, the "armouring" of the ball surface, and also any work hardening of the metal, would be expected to reduce the rate of ball wear at high speeds.

Having regard for the general uncertainties in the data studied, the equations can be simplified to, for all speeds,

$$W_t = KD^{4\cdot 65}(N/N_c)d^{-1\cdot 0}Lt \qquad \ldots . \; (8.13)$$

and

$$W_b = KD^{2\cdot 65}(N/N_c)d^2Lt \qquad \ldots . \; (8.14)$$

An equation given by Mortsell(6), for the rate of metal wear, is

$$W_t' = \left(\frac{8KM_b}{d}\right) \qquad \ldots . \; (8.15)$$

where $M_b$ is the total weight of balls, $W_t'$ is the wear per unit time, $d$ is as before and $K$ is a constant which generally falls between 9 and 16 for Swedish ore but, in extreme cases, between 5 and 23; $W_t'$ being in kilograms per hour and $M_b$ in metric tons.

Since $M_b \propto D^2L$ it follows that, from this equation, the total wear is proportional to $D^2$; not to $D^{4\cdot 65}$ as suggested previously.

It is interesting now to examine equations (8.13) and (8.14) in the light of the published results of the various authorities.

Firstly, it has been suggested by Davis,(9) that the rate of wear of an individual ball is proportional to $d^3$. But Ellis,(10) Prentice,(11) Garms and Stevens,(12) Norman and Loeb,(13) and Norquist and Moeller(14) state that the rate of wear is proportional to $d^2$. Furthermore Bond,(15) gives various figures, such as 2·0, 2·21 and 2·29 whilst Taggart (16) gives 2·0 for cascading balls. Thus, on this point, the results of the present analysis are roughly in agreement with the view of the majority of other workers, but more work is required before this question is settled.

The question of the influence of the mill diameter does not appear to have received more than passing study but the suggestion that the ball wear varies as $D^{2\cdot 6}$, where $D$ is the mill diameter, appears to be reasonable. A serious criticism of all the work cited is that no effort appears to have been made to maintain similarity between the various tests. The most satisfactory type of equation would appear to be equation (8.6) and, whenever possible, the results should be presented in this, or similar, form.

Since, however, the balls are invariably of iron or steel the value of $E_m$ and $\varrho$ may be regarded as sensibly constant, so the equation may be reduced to

$$W_t = KD^{3\cdot5}Lt\,.\,\phi_1(H)\,.\,\phi_3\left(\frac{N}{N_c}\right).\,\phi_4\left(\frac{D}{d}\right).\,\phi_5\left(\frac{b}{D}\right) \times \phi_7(J)\,.\,\phi_8(V) \qquad \ldots . \quad (8.16)$$

An effort has been made to calculate the value of $K$ in this equation, on the basis of the various published work, augmented with reasonable assumed values where necessary, and these values are given in Table 8.1; all values in C.G.S. units.

TABLE 8.1

| Authority | Value of $K$ |
|---|---|
| McLeod(7) test: (1) Chill-cast balls | $1\cdot6\times10^{-8}$ |
| (2) Sand-cast balls | $1\cdot6\times10^{-8}$ |
| (3) Annealed balls | $2\cdot3\times10^{-8}$ |
| Coghill and Devaney(4) | $0\cdot6\times10^{-8}$ |
| Mortsell(6) (pot mill test) | $2\cdot6\times10^{-8}$ |
| Mortsell (from eqn. (8.15) for 6-ft × 6-ft mill) | $1\cdot2\times10^{-8}$ |

The values calculated from equation (8.16) show fairly wide disagreement but this is not surprising since values for many of the parameters have been assumed. It appears, however, that $K = 1\cdot6 \times 10^{-8}$ with an error of about $\pm 50\%$. Thus it appears safe to conclude that the foregoing analysis includes the major factors in the mechanical wear of balls and liners and it is possible that it can form an adequate basis for an extended and improved treatment.

It should also be pointed out that corrosion has an important effect on the rate of liner wear. Thus, if corrosive substances are present, the products of corrosion will be removed from the working surfaces by abrasion and new metal will be constantly exposed to chemical action. The wear arising from this cause will be proportional to the area of metal exposed, but there is insufficient data for an analysis of this problem.

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CHAPTER 9

# THE ROLE OF ADDITIVES IN MILLING

THE term "additive" is applied, in milling practice, to various substances which, when added to the contents of a mill, bring about an improvement in the rate of grinding and, therefore, a reduction in the time of grinding required to attain a specified degree of fineness. Additives could also be introduced in order to attain some special characteristic of the final product. However, such applications are dependent on the purpose of the final product and so are outside the scope of the present chapter. An interesting feature of additives is that the quantity required is generally very small; usually being between about 0·01 % to 0·1 % of the weight of the charge treated.

That additives are of sufficient importance industrially to warrant consideration is shown by recent work of Skaupy,(1) in which it is shown that the rate of milling of iron powder in iso-amyl alcohol is no less than twenty times the rate in dry air. These tests were carried out on a vibration mill, and so might not be directly comparable with a ball mill, but nevertheless the result is significant. In view of the large energy requirements for grinding, especially fine grinding, a saving of even 20 % in milling time could make a substantial saving in milling costs.

In view of the potential importance of these substances it is perhaps surprising that their mode of action is not precisely known. It is impossible to establish from the literature on the subject whether their action arises from improvement in the conditions of crushing by the prevention of a coating of powder on the ball surface, from the prevention of agglomeration in dry milling, or flocculation in the case of wet milling, or by some action within the particle, such as the promotion of the spread of the fissures which occur naturally in all solid materials. It is possible, indeed probable, that all these modes of operation are of importance, either singly or in various combinations, depending on the circumstances and the controlling conditions present.

For dry milling, the additive may be present in solid form, in the form of a solution added to the dry feed, or in a gaseous or vapour state. For wet milling, the additive is present in the water; the concentration generally being less than 1 % of the solid material present.

Before discussing in detail the use of additives in milling it is perhaps relevant to recall what has been said earlier. As shown in Chapter 4, for a substance in which the yield stress and the stress for fracture coincide, the strain energy eventually reappears as heat and the energy associated with the creation of new surface represents a very small fraction of the total energy involved in the crushing operation; this energy being only about 5 % of the total when a particle is comminuted in a simple impact crusher. For very fine powders, however, the surface energy becomes more important and manifests itself in high adsorbtivity and the tendency to agglomeration observed in finely divided substances. Even so, the energy associated with the surface is a small proportion of the total involved. Because of this it is difficult to see how the reduction of the surface energy can significantly affect the energy required for grinding but, nevertheless, surface active dispersing agents are often very effective grinding aids. The reduction of the surface energy of the particle itself is not, however, the only possible explanation of the action of these materials.

A possible direct mechanism for the operation of these substances has been suggested by Rehbinder.(2) Assuming that the molecules of the additive are adsorbed on the surface of the particles, then the cohesive forces which bond operation of solid additivesin dry grinding; for example the improved mill performance obtained when a small quantity of carbon black is added to the cement clinker undergoing grinding. Here again, it is suggested that the function of the additive is in the prevention of cushioning since Schweitzer and Craig(4) have observed that addition of carbon black in proportions as low as 0·08 % is highly effective as a cleaning agent for the surfaces of the balls and mill.

A rather different explanation of the operation of additives is put forward by Bond(5); it again being suggested that ball coating is an important factor. These tests showed, in the first place, that with a soft material, such as limestone, excessive ball coating occurs with powders of much larger particle size than for hard materials, such as quartz. Furthermore the occurrence of coating is practically independent of the character of the grinding media; cast, forged and ball-bearing steel balls showing no significant difference other than that, when new, the smooth hard balls are not subject to coating. Once abrasion has removed the surface, however, these balls show the same behaviour as the others.

According to Bond, the ball coating consists of an innermost layer of particles, of about 0·5–50μ in diameter, wedged into the surface of the ball. The outer layers of particles making up the coating are keyed to this inner layer and are of much larger size. The effect of these layers is to cushion the grinding surface and so to greatly decrease the grinding rate. It is possible that other mechanisms are operative; for example the beneficial results obtained by the use of small quantitites of carbon black as a grinding aid with cement clinker suggests some analogy with electrostatic precipitation. In this application it is known that the addition of a small amount of carbon, generally in the form of unburnt fuel, can bring about the precipitation of a flue dust which otherwise cannot be treated; the function of the carbon apparently being to form a conducting network through the layer of powder by which electrical charges leak away to the collector electrode. Is it not possible, therefore, that some such mechanism is present in this application?

The question of additives will now be considered in more detail; commencing with dry grinding. As previously stated, in dry grinding the additive may be present, at least in theory, in solid, liquid, vapour or gaseous form and some of the most interesting results of the use of these additives are reported from the cement industry.

According to Berry,(6) advantages have been shown by the use of vinsol resin, cod oil, beef tallow, aluminium stearate and glycerol in the grinding of cement clinker. In particular colloidal carbon and two dispersing agents, known as R.D.A. and T.D.A. have proved effective. According to Kennedy,(7) R.D.A., which is arylalkylsulphonic acid, and T.D.A., which is a mixture of triethanol-amino salts and highly purified calcium salts of lignin sulphonic acid, were introduced as dispersing agents and catalysts to improve the properties of cement. These additives were introduced in the form of a solution in water, to the raw clinker feed of the tube mill in the proportion of about 0·06 % by weight of the charge. Apart from the improvement in the setting properties of the cement, the output of the mill, for the same specific surface, was reported to be increased by 30–40 %. The researches of Schweitzer and Craig(4) into the grinding of cement clinker, in a laboratory ball mill, with carbon black as a grinding aid show a continuous increase in the rate of grind, or fineness attained in a given time, for additions up to about 1 % by weight of the feed. The rate of grinding thereafter fell with increasing amounts of the additive.

These results, which are summarized in the curve of Fig. 9.1, show that the time required to grind to 85 % through 325 mesh is reduced by 35 % when compared with that required when no additive is used. The curve also shows that the optimum quantity of additive is fairly definite; small deviations from the optimum quantity having appreciable effect. This conclusion appears to be true for most additives.

As previously noted, quantitites of additive as low as 0·08 % by weight of the solid present had a marked effect on the cleanliness of the mill surfaces, without producing noticeable colour deterioration of the product. The rate of grinding is not, however, markedly improved by this small quantity of additive.

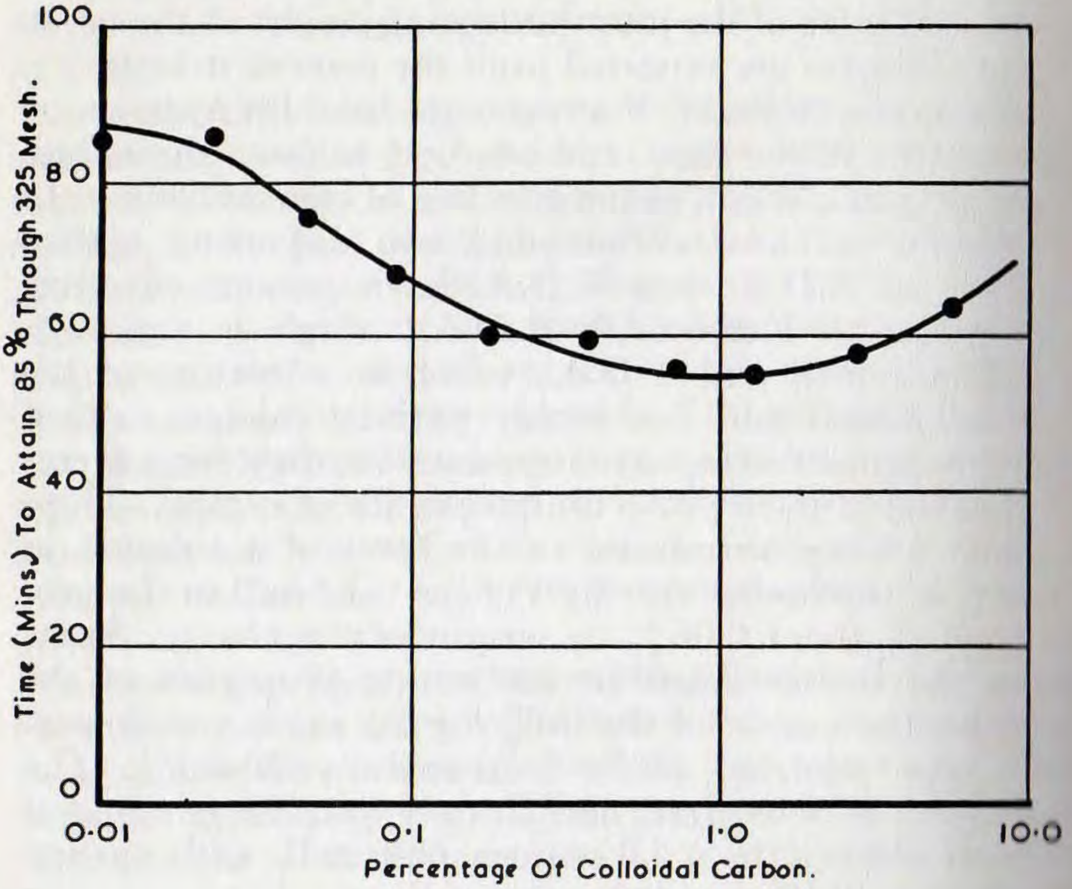

FIG. 9.1

The work of Bond(5) shows that the addition of a small amount of water, 0·88 % by weight, brought about a reduction in the coating of the balls. A solution of T.D.A., at about 0·13 % by weight of the feed, however, brought about complete inhibition of ball coating. It was therefore concluded that the action of the additive in preventing coating of the ball charge was more important than effects arising from adsorption on the surface of the particles. This view is supported by the fact that the small amounts of additive which are sufficient to bring about a marked improvement in the rate of grinding, are sufficient to allow the formation of a monomolecular layer on less than 20% of the product surface. It is also supported by the fact that even large changes in the surface energy would produce but small proportional changes in the energy demanded by the whole grinding process (since the surface energy represents a very small fraction of the total energy demand of the process). It is also significant that the solid additives, mentioned earlier, all tend to inhibit ball coating; probably by imparting a greasy surface to the grinding media since they are all known to exhibit marked lubricating properties.

The effect of very small quantitites of water in preventing ball coating is known in other applications. For example, Manson(8) states that it is well known in the paint industry that, in the grinding of dry-process enamels, the addition of as little as 0·0625% of water can be effective in the prevention of ball and liner coating. Furthermore, the coating can also be removed from badly coated balls, under running conditions, by the addition of a few wet balls to the mill charge. Bond(5) suggests that selective hydration of the particles at the ball surface might be the reason for the inhibition of coating by small quantities of water. Such a suggestion is, at the most, tentative.

The reason for such preferential hydration at the surface of the balls is not clear, but it must be remembered that, at the moment of impact, there is a considerable amount of energy dissipated and, doubtless, some of this could be used to bring about various chemical, or physicochemical, reactions which demand an input of energy. This possibility is supported by an observation by Pryor,(9) who found that copper ore was changed to a compound of a higher energy level at points where the balls of the mill rubbed the ore without breaking it.*

---

* One of the authors (H.E.R.) has a piece of this ore, presented to him by Mr. Pryor, which clearly shows this transformation at a number of points where contact has been made with the balls.

There are other examples of the increase in the rate of grinding brought about by the use of additives in dry milling. For example, Waeser(10) states that both R.D.A. and ammonium salts are effective in the grinding of graphite and that oleic acid is an effective additive for the grinding of zinc blende. Götte and Ziegler(11) quote interesting results as to the milling of cement clinker, marble and glass in atmospheres of various polar and non-polar substances. The tests were carried out in a vibrating mill, the additive being introduced in vapour form into the previously evacuated mill body containing the feed. The polar additives investigated were acetone, nitromethane and water; the non-polar additives being carbon tetrachloride, benzol, hexane and petroleum ether.

These results can be summarized as follows:

(1) In the milling of clinker, the improvement brought about by the use of non-polar additives in place of an atmosphere of dry air, decreases with increasing fineness and increasing time of grind; the average increase in relative fineness being 10–20 %. The polar additives show an increasing effectiveness with time of grind and fineness of powder; the most effective being acetone which led to improvements of 10–50 % in the rate of grinding, relative to that carried out in dry air.

(2) Marble showed the same general characteristics but, in this case, water vapour, which led to a 60 % increase in speed of grinding compared with a dry atmosphere was the most effective additive.

(3) For the grinding of quartz, polar and non-polar atmospheres led to about the same increase in the rate of grind.

(4) Glass proved anomalous in that, whilst non-polar additives effected some small increase in the rate of grind, polar additives produced a very definite *decrease* in the grinding rate. No definite explanation

of this anomaly is forthcoming but it is possible that it might be connected with the strong adsorptive properties of finely divided glass powder.

Whilst it is realized that the tests of Götte and Ziegler are not strictly relevant, in so far that they were carried out in a vibration mill, it is considered that they are of interest in that they are, as far as is known to the authors, the only tests carried out with a controlled atmosphere consisting of a surface active agent in vapour form. The method may well have applications for certain specialized batch dry-milling operations.

Turn now to the use of additives in wet milling. According to Berry,(6) silicates, phosphates and arylalkylsulphonic acid are often used as additives in wet milling; the paint and colour industry, especially, offering a very wide field for the use of such agents (Anon.(12)).

Probably the most thorough investigation into the effects of additives in wet milling is that reported by von Szantho(13) In this work, the effect of varying percentages of flotation additives and of oleonic acid on the wet milling of quartz and limestone, in a laboratory batch ball mill, was investigated. In each case the characteristic product particle size, *d*, resulting from grinding for 30 minutes was estimated from the size frequency curves. The results, showing this characteristic particle size plotted against the percentage concentration by weight of the additive, are shown in Fig. 9.2. It is seen that, in each case, there is an optimum percentage beyond which the grinding rate decreases with any increase in the quantity of additive.

Von Szantho considers that the effect of the additive is primarily due to the lowering of the surface energy of the particles, arising from the adsorption of the additive, with consequent widening of the elementary cracks and fissures on the surface of the particles. As discussed previously, it is implicit in this conception of the operation of additives that their effect should become progressively greater with increasing fineness of grind. The undersize curves shown in Fig. 9.3 do in fact show that the effect of the additive is

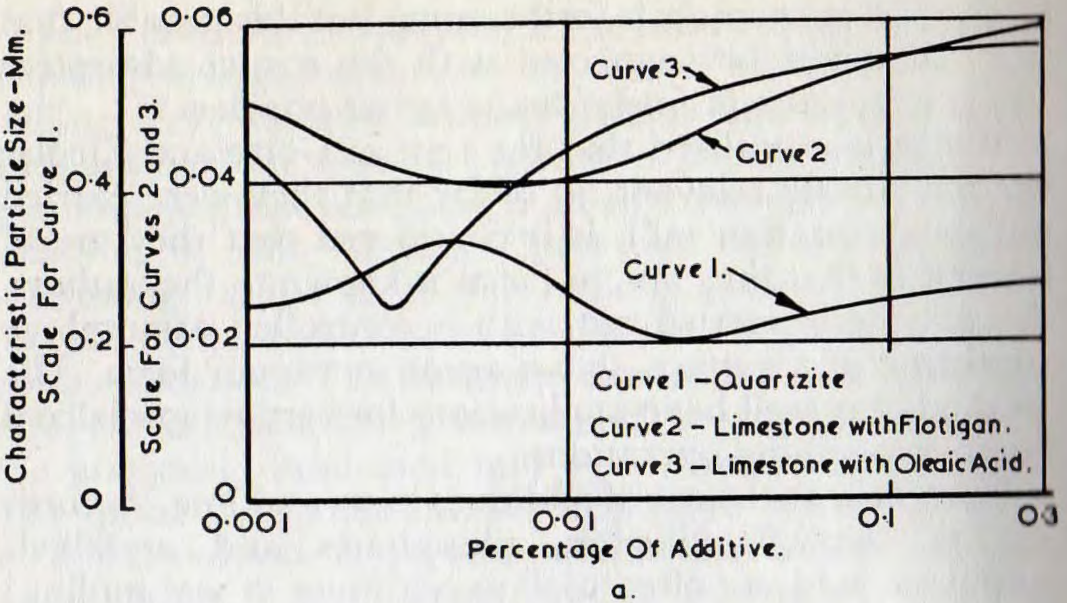
Characteristic Particle Size – Mm.
Scale For Curve I.
Scale For Curves 2 and 3.
0·6
0·4
0·2
0
0·06
0·04
0·02
0
Curve 3.
Curve 2
Curve I.
Curve I – Quartzite
Curve 2 – Limestone with Flotigan.
Curve 3 – Limestone with Oleaic Acid.
0·001
0·01
0·1
0·3
Percentage Of Additive.
a.

FIG. 9.2

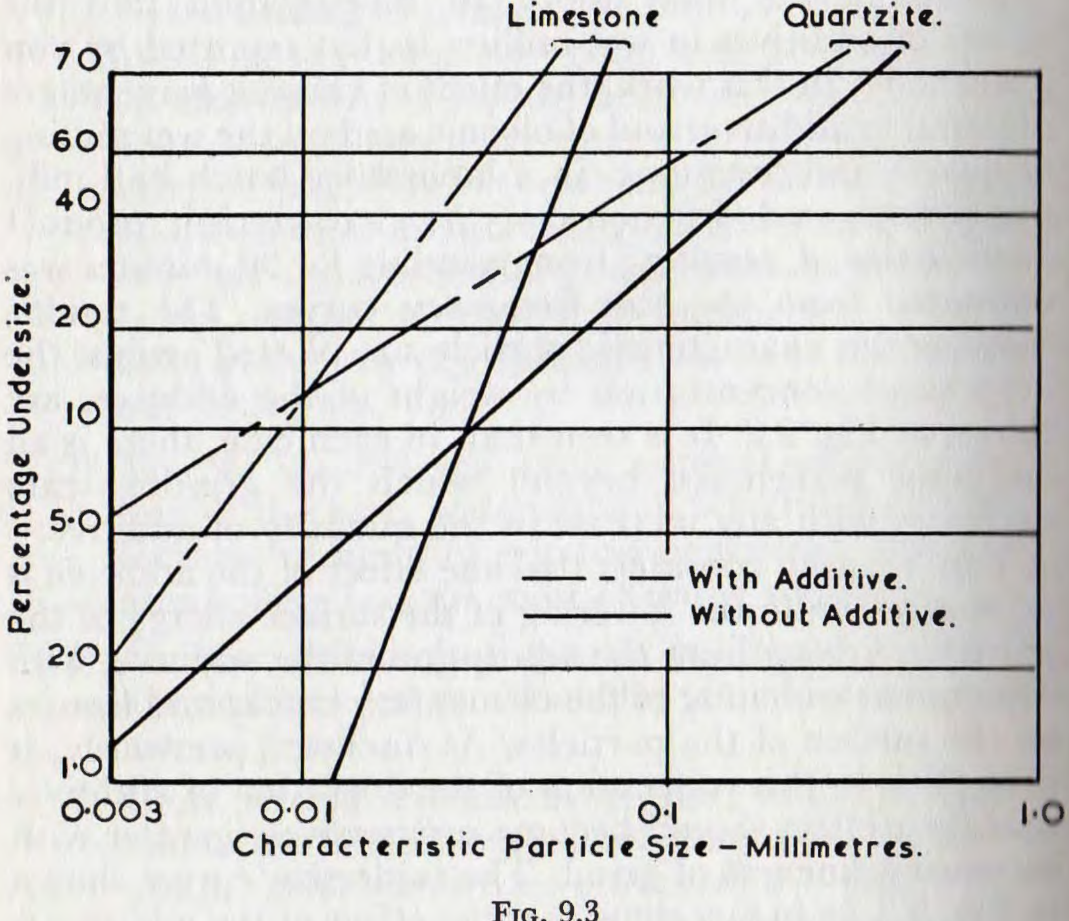
Limestone
Quartzite.
70
60
40
20
10
5·0
2·0
1·0
Percentage Undersize.
With Additive.
Without Additive.
0·003
0·01
0·1
1·0
Characteristic Particle Size – Millimetres.

FIG. 9.3

greater at the smaller particle sizes (for particles below about 60μ).

The decrease in the rate of grinding observed when the concentration of additive exceeds a certain value is tentatively explained on the basis of the lubricating effect of these substances at high concentrations. This explanation is supported by the data of Fig. 9.4 and Fig. 9·5, in which curves of surface produced and coefficient of friction are plotted against concentration of additive. These curves

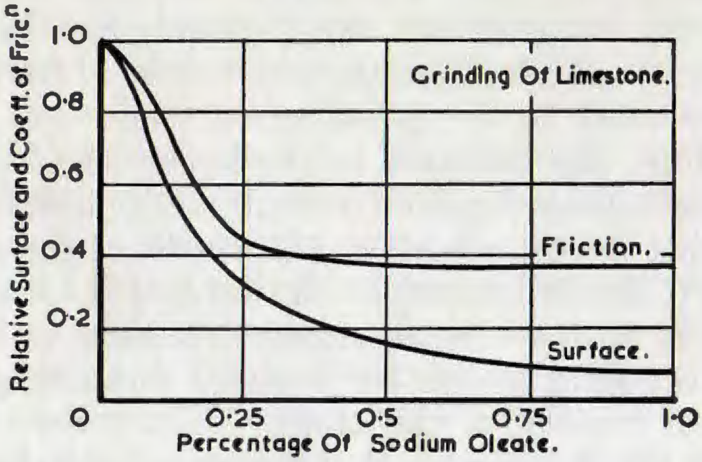

FIG. 9.4

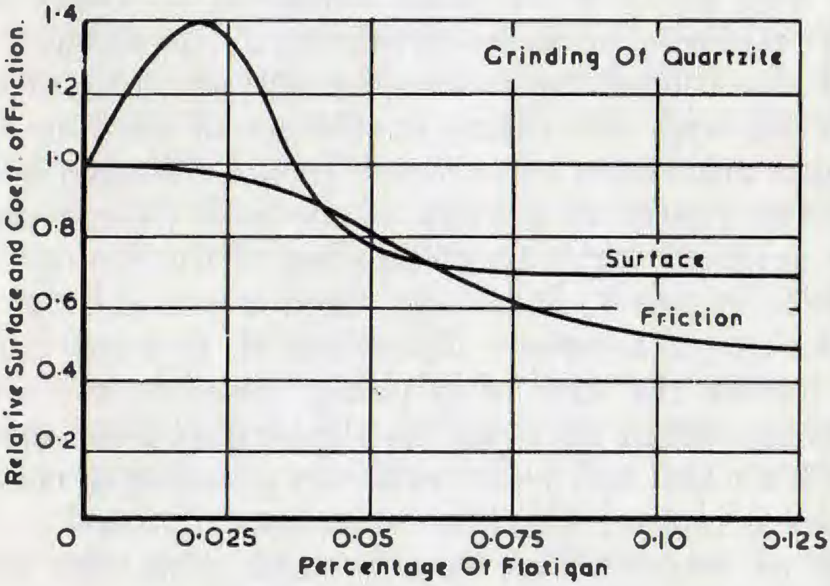

FIG. 9.5

show that for concentrations of additive for which the coefficient of friction is high the rate of grinding is high, but when the coefficient of friction decreases the rate of grinding diminishes also; the curves of coefficient of friction and rate of grinding being closely similar. Thus, von Szantho concludes that the coefficient of friction is of major importance in the grinding rate, and that, at high concentrations, the decrease in coefficient of friction outweighs the advantages gained from the reduction in surface energy arising from the surface adsorption of the additive. This result is also of interest in the light of the contents of Chapter 7, in which it is suggested that surging in a mill might be brought about by a small reduction in the coefficient of friction of the charge. Thus, the curves of Fig. 9.4 and Fig. 9.5 suggest that the use of additives might reduce the coefficient of friction by 40–50% and this could well prove more than sufficient to cause a mill, which otherwise performs satisfactorily, to surge.

It is also interesting to enquire why the rate of milling should fall with decreasing coefficient of friction. Clearly this must arise from a reduction in the effective height of fall of the centre of gravity of the mill charge and this would probably arise from slipping of the charge on the mill shell. If this is, in fact, so, then it would be expected that the use of adequate lifters would, to some extent at least, restore the rate of grinding. Possibly there is also some relationship between the slipping and the increased rate of liner and ball wear, with wet grinding as compared with dry grinding, which is sometimes reported.

It is of interest that von Szantho cites the work of Rehbinder(2) and of von Engelhardt(14) in support of his conclusions. Rehbinder showed a correlation between the surface hardness of materials and their resistance to scratching with a fine point. It was also shown that there is a progressive decrease in the resistance to scratching with increasing quantities of additive adsorbed on the surface of the body, there being a close correlation between the resistance to scratching and the adsorption isotherm, with a minimum of resistance to abrasion being reached under conditions of complete adsorption. As much as 70 % decrease in surface hardness was measured in this way, the maximum decrease corresponding to the greatest difference in polarity between the medium and the surface. It is suggested by Rehbinder that non-polar additives would be the most effective for hydrophilic materials, and polar additives should be used for hydrophobic crystals.

Engelhardt reports the work of Obreimoff,(15) who measured the elastic work done in separating the lamina of mica and found that this was numerically equal to the theoretical surface energy of the new surface formed. When splitting of the lamina was carried out in the presence of surface active media, however, the surface exposed, for a given amount of strain energy, increased in proportion to the decrease in the surface energy caused by the adsorption. Engelhardt further conducted tests on the abrasive grinding of quartz with a silicon carbide disc, in the presence of various additives. It was found that polar and dipolar additives were very effective in reducing the resistance to abrasion. In particular small quantities of octanol or butyric acid in benzol or sodium silicate in water were effective in reducing wear resistance.

Skaupy(1) has recently reported some outstanding results obtained from the milling of iron powder in various fluid media. Although the tests were carried out in a vibrating mill, they are included here on the grounds of their importance. It was found that the rate of grinding of iron was five times as great in glycol, and no less than twenty times as great in iso-amyl alcohol as in dry air. The tests revealed that dipolar additives were especially effective as grinding aids, as would be expected from surface energy considerations alone.

Although, in view of the more marked effect, the evidence points to a lowering of the surface energy as being an important factor in the operation of additives in wet milling, the evidence in the case of dry milling is more confused. The effectiveness of various materials with proshow that for concentrations of additive for which the coefficient of friction is high the rate of grinding is high, but when the coefficient of friction decreases the rate of grinding diminishes also; the curves of coefficient of friction and rate of grinding being closely similar. Thus, von Szantho concludes that the coefficient of friction is of major importance in the grinding rate, and that, at high concentrations, the decrease in coefficient of friction outweighs the advantages gained from the reduction in surface energy arising from the surface adsorption of the additive. This result is also of interest in the light of the contents of Chapter 7, in which it is suggested that surging in a mill might be brought about by a small reduction in the coefficient of friction of the charge. Thus, the curves of Fig. 9.4 and Fig. 9.5 suggest that the use of additives might reduce the coefficient of friction by 40–50% and this could well prove more than sufficient to cause a mill, which otherwise performs satisfactorily, to surge.

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Although, in view of the more marked effect, the evidence points to a lowering of the surface energy as being an important factor in the operation of additives in wet milling, the evidence in the case of dry milling is more confused. The effectiveness of various materials with pronounced lubricating properties such as carbon black, as an aid in the grinding of cement clinker, is much easier to explain on the basis of the inhibition of ball coating by means of the deposition of a layer of carbon on the balls. Furthermore, the small quantities of water which have been observed to remove the coating from a badly coated mill lends support for the idea put forward by Bond⁽⁵⁾ that, in this case, the particles are removed from the surface by a process of selective hydration, a surface phenomenon. However, the small quantities of water which are effective suggests that the adsorption occurs at the ball surfaces only. It thus appears that the surface phenomena involved are complex and that present knowledge is insufficient to give more than a very imperfect picture of the mechanism which is operative.

In conclusion, it is fair to say that any process of crushing must be influenced to a large extent by the nature of the atmosphere in which the operation is carried out. There is ample experimental evidence that in many cases, the energy or time of grind to a given fineness, and also the ultimate fineness of grind attainable, can be favourably modified by the introduction of a suitable additive into the mill. In view of the very large energy requirements for grinding, and in particular fine grinding, and also the low theoretical efficiency of the grinding operation as at present carried out, it would appear that the use of milling additives could effect substantial economies. It is also clear that the subject has not yet received the attention which its importance warrants, and it appears to be a field in which much interesting research remains to be done.

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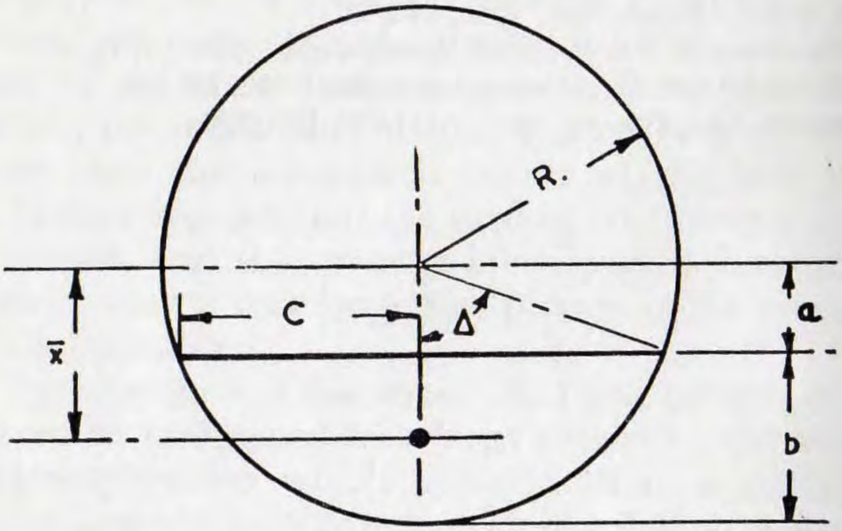
# APPENDIX I



MILL FILLING PARAMETERS

| $\Delta°$ | $J$ | $C/R$ | $a/R$ | $b/R$ | $\bar{x}/R$ |
|---|---|---|---|---|---|
| 0 | 0·000 | 0·000 | 1·000 | 0·000 | 1·000 |
| 15 | 0·003 | 0·259 | 0·966 | 0·034 | 0·960 |
| 30 | 0·028 | 0·500 | 0·866 | 0·134 | 0·950 |
| 45 | 0·090 | 0·707 | 0·707 | 0·297 | 0·810 |
| 60 | 0·194 | 0·866 | 0·500 | 0·500 | 0·700 |
| 75 | 0·335 | 0·966 | 0·259 | 0·741 | 0·572 |
| 90 | 0·500 | 1·000 | 0·000 | 1·000 | 0·430 |
| 105 | 0·665 | 0·966 | −0·259 | 1·259 | 0·288 |
| 120 | 0·804 | 0·866 | −0·500 | 1·500 | 0·173 |
| 135 | 0·910 | 0·707 | −0·707 | 1·707 | 0·091 |
| 150 | 0·972 | 0·500 | −0·866 | 1·866 | 0·028 |
| 165 | 0·997 | 0·259 | −0·966 | 1·966 | 0·004 |
| 180 | 1·000 | 0·000 | −1·000 | 1·000 | 0·000 |

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